Critical Thinking in Aquifer Test Interpretation

Interpretation of pumping well drawdowns

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Last update: April 28, 2025

1. Notes on the interpretation of pumping well drawdowns

2. Additional readings

   - Theis et al. (1963)
   - Bradbury and Rothschild (1985)
   - Jacob (1946)
   - Rorabaugh (1953)
   - Bierschenk (1964)

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Overview

The drawdowns in a pumping well can and should be interpreted for every pumping test. Key to the reliable interpretation of the drawdown data from a pumping well is an appreciation that the drawdowns in a pumping well reflect more than just the effects of head losses in the formation. In these notes the interpretation of pumping well drawdowns is built up gradually in complexity. The notes begin with the characterization of the components of the drawdown in a pumping well. Models are then discussed to support the inference of well characteristics and aquifer properties.

Outline

1. Definition of the drawdown in a pumping well
2. Components of the drawdown in a pumping well
3. Representation of head losses in the formation
4. Representation of the additional head losses across a skin zone
5. Representation of the additional head losses due to partial penetration
6. Representation of the additional head losses due to turbulent near or within the well
7. Diagnosis of additional well losses
8. Further investigation of additional well losses
9. Preliminary estimation of the transmissivity from the specific capacity of a pumping well
10. Complete transient analysis with the incorporation of additional well losses
11. Step tests
12. Interpretation of step tests: Steady-state analysis
13. Interpretation of step tests: Transient analysis
14. Synthesis of pumping well and observation well drawdowns
15. Key points
16. References

1. Definition of the drawdown in a pumping well

The drawdown in a pumping well is defined as the difference between the water level in a well under nonpumping conditions and the water level observed in the well when it is pumping.[1] The drawdown is illustrated schematically in Figure 1, but the definition is slightly more general than what is suggested in the figure. The non-pumping level in an aquifer is never a completely flat surface, nor will it remain constant through time. Data from a pumping test conducted in Portland, Oregon, are shown in **Error! Reference source not found.** to highlight that the drawdown is correctly interpreted as the difference between the pumping level and the level that would be observed in the absence of pumping.

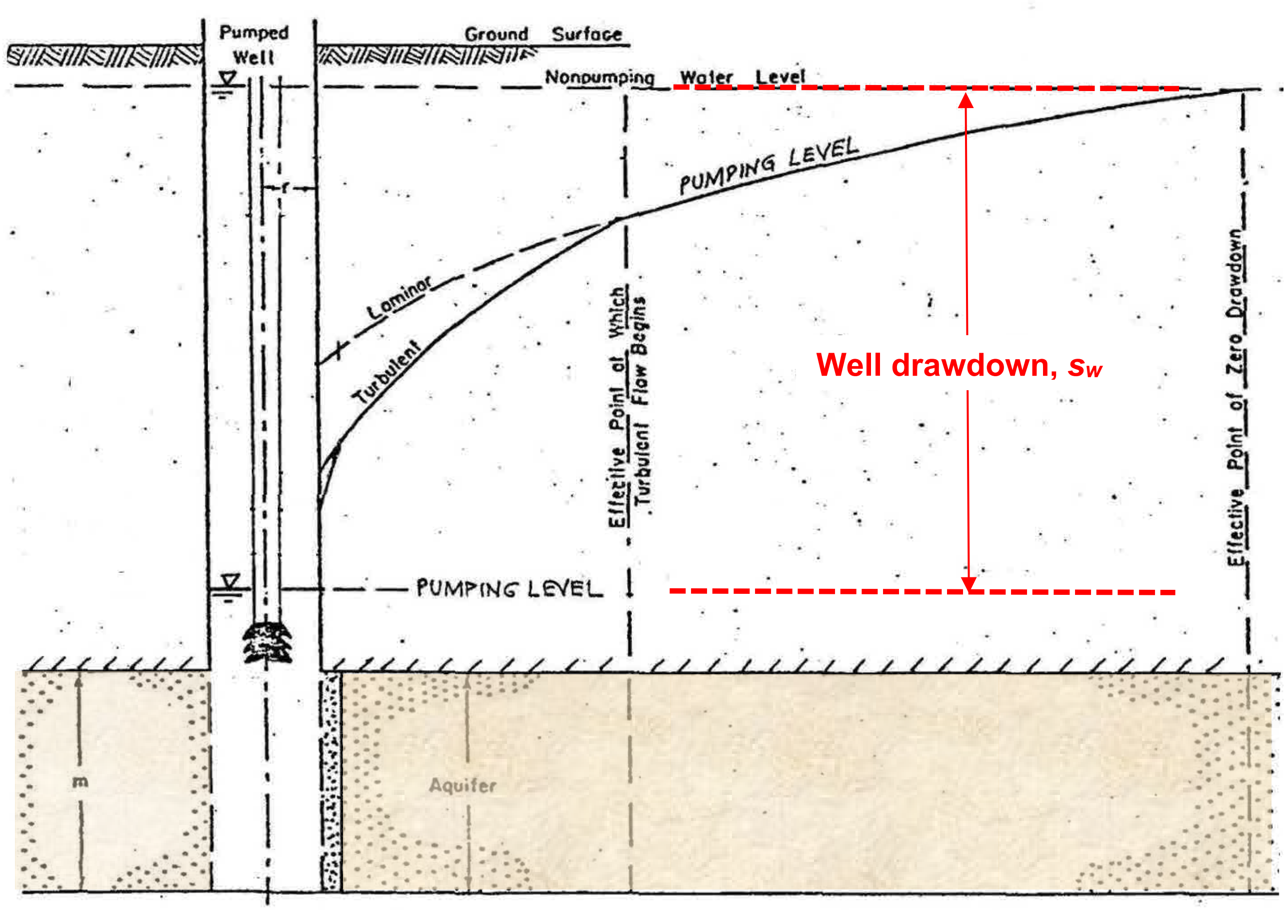


Figure 1. Drawdown in a pumping well
Adapted from Bruin and Hudson (1955)

[1] For simplicity, we will refer to "water level" rather than potentiometric level or hydraulic head.

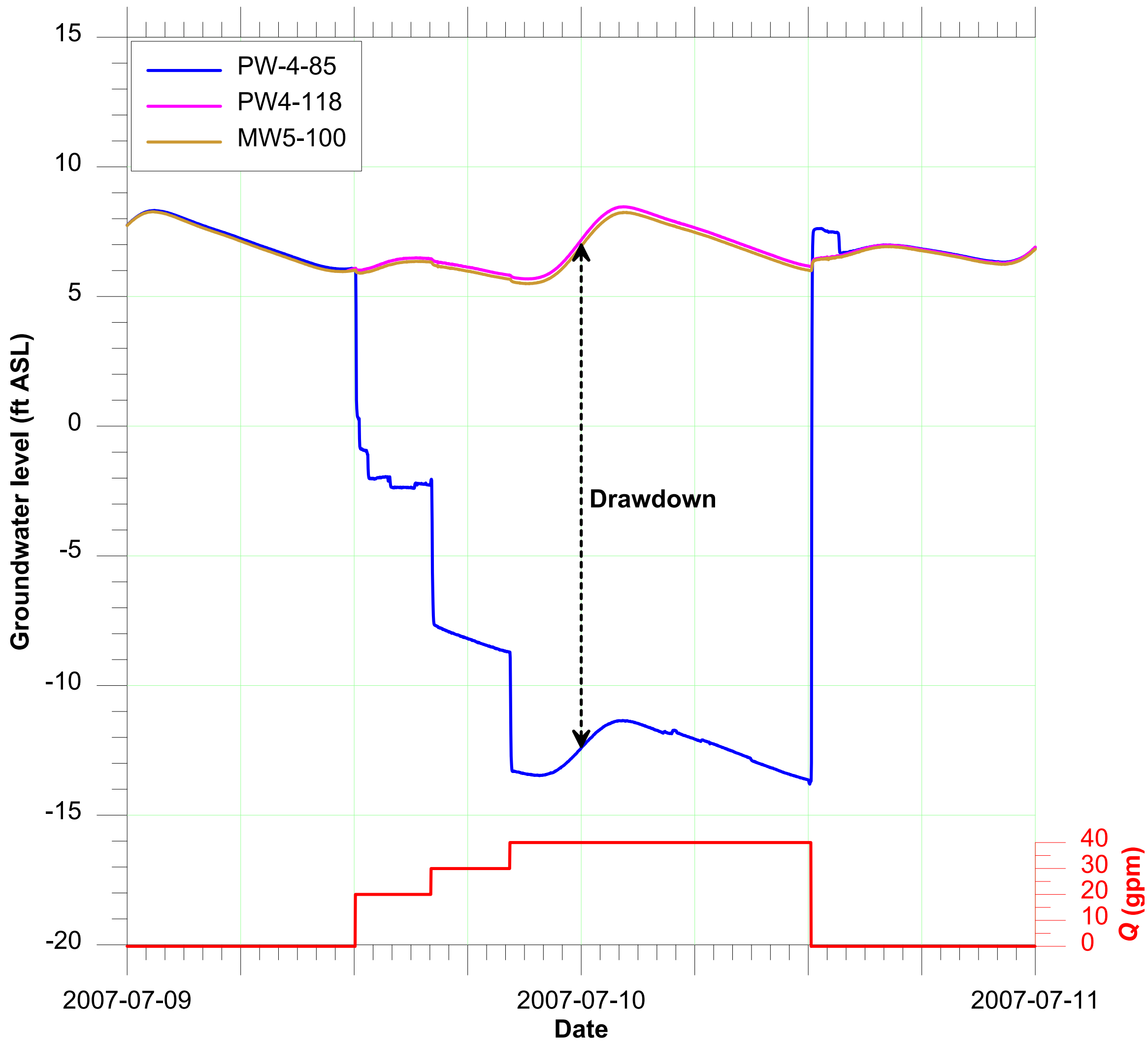


Figure 2. Interpretation of the drawdown in pumping well PW-4-85

2. Components of the drawdown in a pumping well

To support the interpretation of pumping well drawdowns it is important to first understand the components of the drawdown. Following the general approach of Walton (1962, 1970), the total drawdown is idealized as consisting of five components. Referring to Figure 3 and moving inwards from the formation to the inner casing, the head losses are:

1. s_a: the head loss due to *laminar* flow in the formation;
2. s_t: the additional head loss due to *turbulent* flow in the formation;
3. s_s: the additional head loss across a zone of reduced permeability around the well;
4. s_e: the additional head loss across the well screen; and
5. s_c: the additional head loss within the well casing itself.

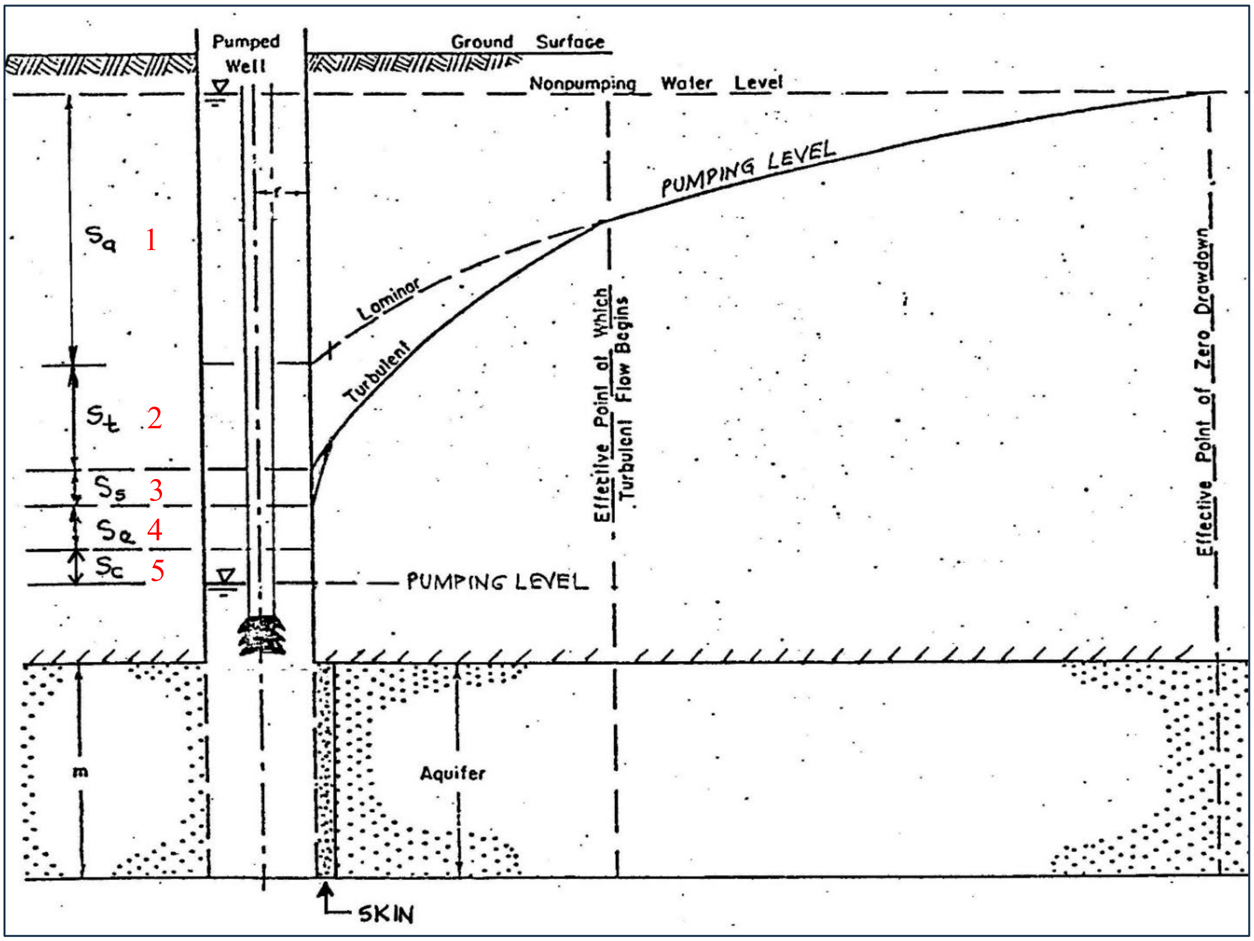


Figure 3. Components of drawdown in a pumping well

The components of the total drawdown in a pumping well are defined below.

1. s_a : Head loss due to *laminar* flow in the formation

The head losses due to laminar flow in the formation arise from friction losses as water is transmitted through the formation towards the pumping well. These head losses depend on the duration of pumping, the properties of the formation (transmissivity and storage coefficient), and the construction of the well (radius and extent of penetration).

2. s_t : Additional head loss due to *turbulent* flow in the formation

If the flow rate is sufficiently high, there may be additional head losses in the formation due to turbulent flow. If a well screened in a porous medium has been designed properly, there should be little possibility of turbulent flow in the formation. However, in fractured-rocks, pumping may induce velocities that are sufficiently high that flow is no longer laminar. Atkinson and others (1994) present an excellent treatment of turbulent flow in discrete fractures.

3. s_s: Additional head loss across a zone of reduced permeability around the well

Regardless of how carefully a well may be drilled, there is always the possibility that a zone of disturbed material may be created around it. The zone of disturbed material is usually referred to as a "skin", and the additional head losses due to its presence are referred to as a "skin effect". Skin effects may arise from the use of drilling mud in porous media, or by the closing off of fractures in rock. Skin effects may be mitigated to a certain extent by proper well development following drilling.

4. s_e: Additional head loss across the well screen

Head losses due to the flow of water across the well screen arise from the constriction in the flow as it passes through the openings of the well screen. These losses are generally referred to as entrance losses. If a well screen has been designed properly, these losses should not be significant. However, they may evolve through time if bacterial growth or mineral precipitates clog the well screen.

5. s_c: Additional head losses within the well itself

Additional head losses may occur within the well itself, due, for example, to turbulence arising from the constrictions around the pump appurtenances.

Total drawdown in the pumping well

The head losses in a pumping well are assumed to be additive. That is, the total drawdown in a pumping well is assumed to be the sum of drawdowns due to each of the components:

$$s_w(t) = s_a + s_t + s_s + s_e + s_c \quad (1)$$

Some researchers have attempted to quantify some or all of the five components of the drawdown indicated in Equation (1) (see for example Barker and Herbert, 1992a,b; and Atkinson et. al., 1994). However, on a practical level, it is generally not feasible to distinguish between all of them. A simplified model will be discussed in these notes, distinguishing only between the laminar head losses in the formation, additional head losses across a skin zone, and additional turbulent head losses:

$$s_w(t) = s_{formation} + \Delta s_{skin} + \Delta s_{turbulence} \quad (2)$$

The representation of these lumped components of the drawdowns are discussed in the following sections of the notes, starting with the representation of head losses in the formation.

3. Representation of head losses in the formation

The component of the drawdown due to head losses in the formation is shown schematically in Figure 4. The head losses in the formation are the difference in the groundwater level in the aquifer under non-pumping conditions and the level at the outside edge of the well screen or borehole at any subsequent time:

$$s_{formation}(t) = h(r_w)_{non-pumping} - h(r_w, t)_{pumping} \tag{3}$$

Here r_w is the effective radius of the pumping well and t is the elapsed time since the start of pumping. The effective radius is frequently assumed to be the outside radius of the borehole. The key aspect of formation losses is that, for either steady or transient flow, if flow in the formation is laminar, the change in the water level (drawdown) should be a linear function of the flow rate from the formation:

$$s_{formation}(t) = Q_{formation} \times F(r_w, t) \tag{4}$$

Here $F(r_w,t)$ denotes a particular aquifer model. When pumping starts the initial water withdrawn from the well is taken from the well casing (wellbore storage). Some time will be required until the flow rate from the formation ($Q_{formation}$) is equal to the pumping rate from the well (Q).

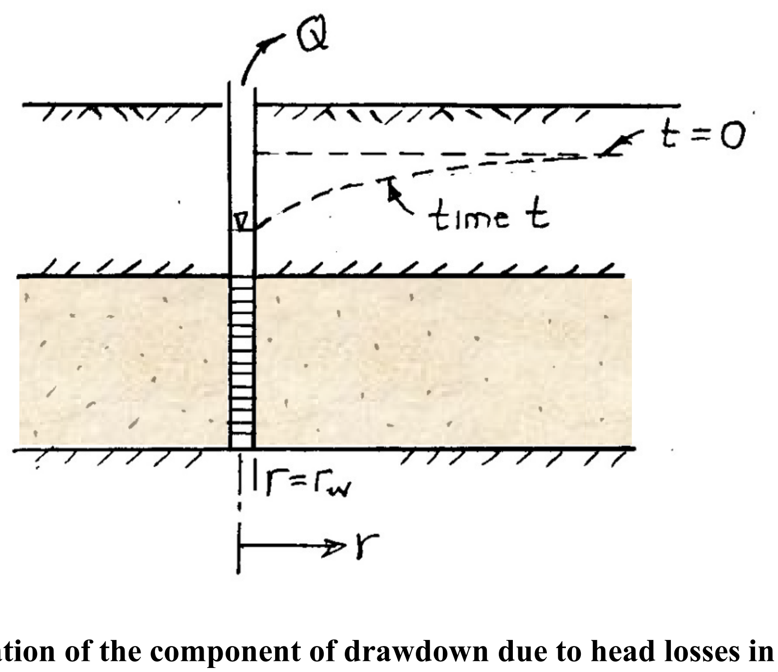


Figure 4. Idealization of the component of drawdown due to head losses in the formation

The Theis (1935) solution is invoked frequently to represent the component of the total drawdown due to head losses in the formation is given by:

$$s_{formation}(t) = \frac{Q_{formation}}{4\pi T} \times W(\frac{r_w^2 S}{4Tt}) \tag{5}$$

Here T denotes the transmissivity of the formation, S the storativity (confined storage coefficient) and t is the elapsed time. The term W denotes the Theis well function (the exponential integral). Application of the Theis solution assumes that the aquifer is extensive, uniform, isotropic, perfectly confined and pumped by a fully penetrating well. The drawdown due to laminar head losses in the formation are evaluated at the effective radius of the well.

As shown in Figure 5, for all but the earliest times after the start of pumping, drawdowns calculated with the Theis well function and Cooper and Jacob (1946) approximation are indistinguishable. Therefore, as a first approximation for the drawdown due to head losses in the formation we can use:

$$s_{formation}(t) = \frac{Q_{formation}}{4\pi T}\left[-0.5772 - ln\left\{\frac{{r_w}^2 S}{4Tt}\right\}\right] \tag{6}$$

This can be expanded as:

$$s_{formation}(t) = \frac{Q_{formation}}{4\pi T}\left[ln\{EXP\{-0.5772\}\} - ln\left\{\frac{{r_w}^2 S}{4Tt}\right\}\right]$$

Making use of the properties of the log function:

$$s_{formation}(t) = \frac{Q_{formation}}{4\pi T} ln\left\{EXP\{-0.5772\}\frac{4Tt}{{r_w}^2 S}\right\}$$

$$= \frac{Q_{formation}}{4\pi T} ln\left\{\frac{2.246\, Tt}{{r_w}^2 S}\right\}$$

Changing to base 10 logarithms:

$$s_{formation}(t) = \frac{Q_{formation}}{4\pi T} 2.303\, log_{10}\left\{\frac{2.246\, Tt}{{r_w}^2 S}\right\} \tag{7}$$

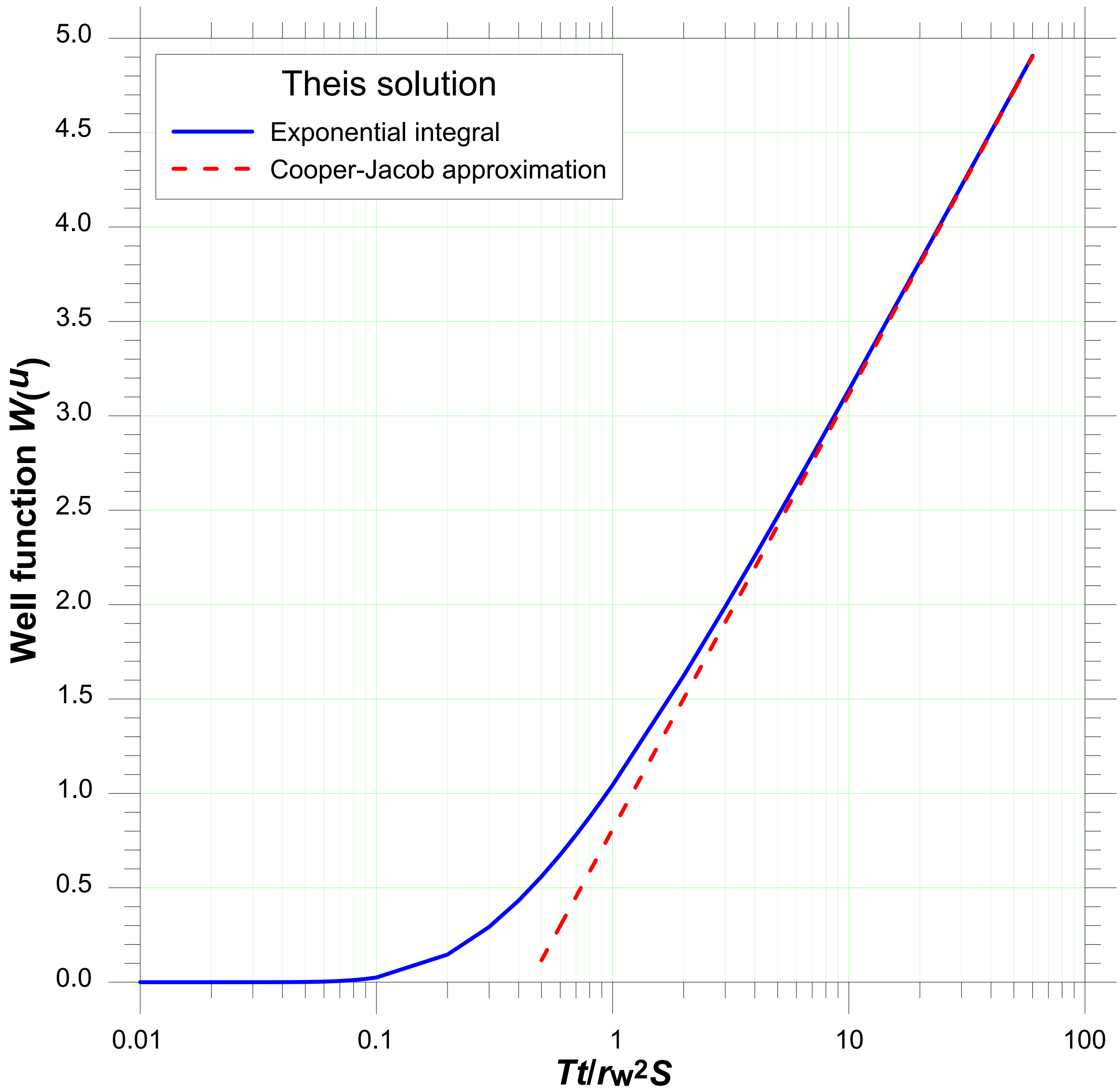


Figure 5. Theis well function and its approximation

4. Representation of the additional head losses across a skin zone

Drilling and installing a pumping well generally cause some alteration in the properties of the formation around the wellbore. The zone of altered material is referred to as the *skin*. If the hydraulic conductivity of the skin is reduced relative to the formation, there will be additional head losses across the skin as shown schematically in Figure 6. The distance from the center of the well to the outer edge of the skin is designated r_s.

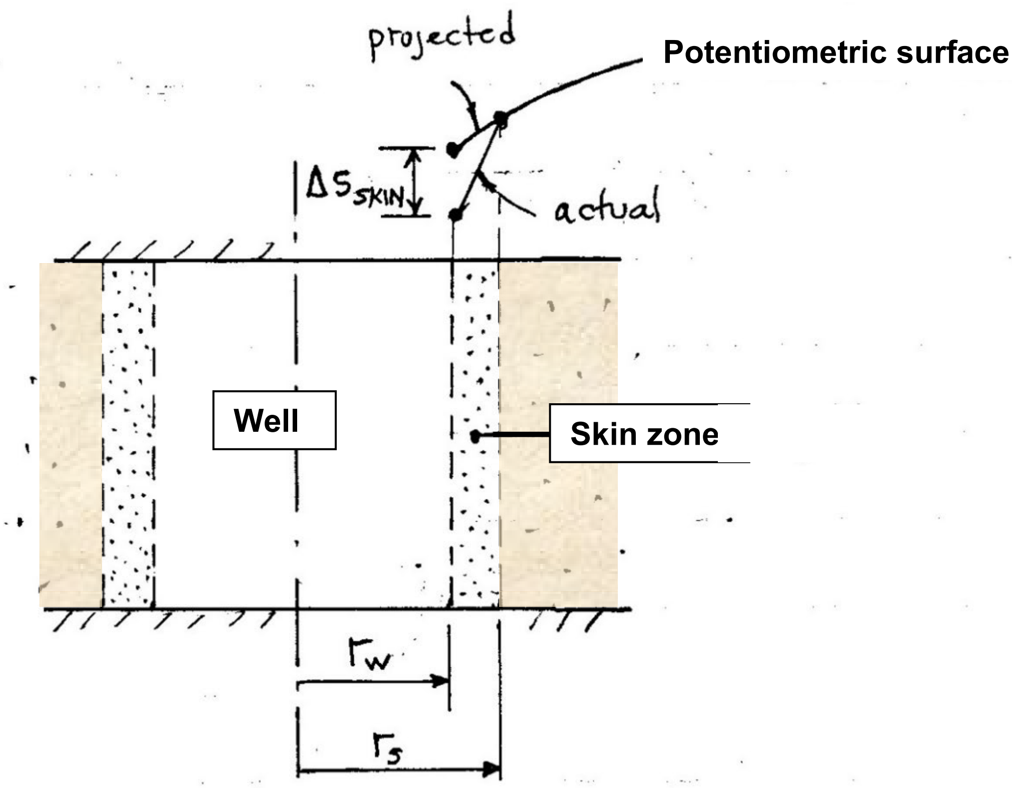


Figure 6. Schematic cross-section of a well surrounded by a skin zone

There are two key aspects of skin losses:

- Skin losses are established relatively quickly after pumping starts; and
- Skin losses are proportional to the pumping rate.

Ramey (1982) proposed that the effects of a zone of damaged material around the pumping well could be represented by a constant additional drawdown:

$$\Delta s_{skin} = \frac{Q}{4\pi T} 2S_w \tag{8}$$

Here S_w is referred to as the *dimensionless skin factor*.

Assuming that there is no storage within the skin zone it is possible to derive an analytical expression for the dimensionless skin factor (Hawkins, 1956):

$$S_w = \left(\frac{K-K_s}{K_s}\right) ln\left\{\frac{r_s}{r_w}\right\} \tag{9}$$

Here K and K_s are the hydraulic conductivities of the formation and the skin, respectively, and r_s is the radius of the skin. Equation (9) can be rearranged to read:

$$S_w = \left(\frac{K_f}{K_s} - 1\right)\ ln\left\{\frac{r_s}{r_w}\right\} \tag{10}$$

This definition is presented as Eq. 2.10 in the classic petroleum engineering text of Earlougher (1977). In this form it is clear why petroleum engineers use the terminology “positive skin” to denote the effect of a reduced permeability of the skin, and “negative skin” to denote the effect of an increased permeability of the skin relative to the formation. In practice, we cannot estimate the extent of the skin zone or isolate its properties. Therefore, S_w is generally treated as a lumped parameter. As will be shown in a subsequent section of these notes, the presence of a skin can frequently be inferred from the estimation of a non-physical storage coefficient.

5. Representation of the additional head losses due to partial penetration

Additional head losses occur when a pumping well does not penetrate the full thickness of an aquifer. The conceptual model of a partially penetrating well is shown in Figure 7.

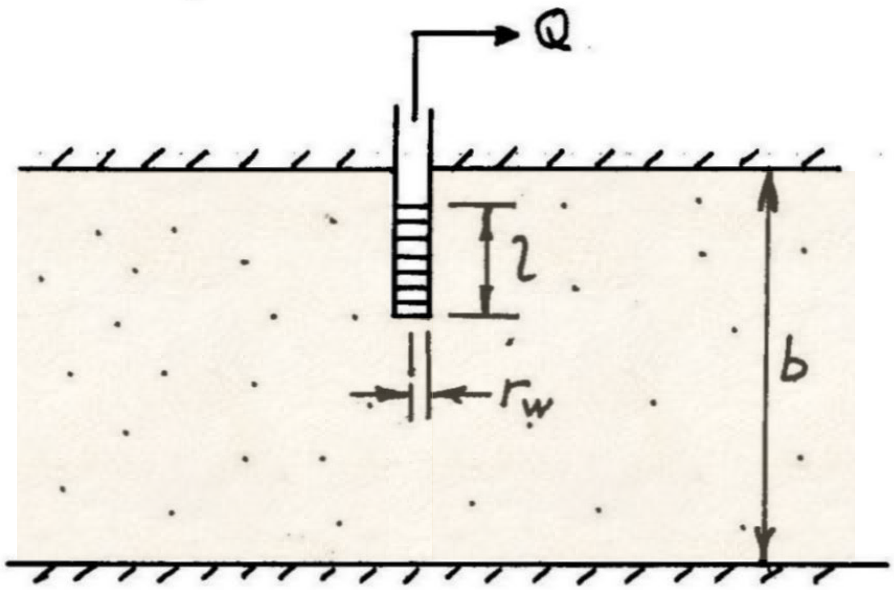


Figure 7. Conceptual model for a partially penetrating well

Rigorous analyses of flow to partially penetrating pumping wells suggest that the additional head losses caused by partial penetration are established relatively quickly and are directly proportional to the pumping rate (Hantush, 1961). Therefore, they have the same general form as skin losses. The losses due to partial penetration are written in terms of a *pseudo-skin coefficient, S_{pp}*:

$$\Delta s_{pp} = \frac{Q}{4\pi T} 2S_{pp} \tag{11}$$

Several approaches have been developed to estimate the additional head losses due to partial penetration. Brons and Marting (1961) developed a simple approach that in our experience closely approximates results obtained with more elaborate calculations:

$$S_{pp} = \left(\frac{b-l}{l}\right)\left[ln\left\{\frac{b}{r_w}\right\} - G\left(\frac{l}{b}\right)\right] \tag{12}$$

Here *b* is the aquifer thickness, *l* is the length of the well screen, and $G\left(\frac{l}{b}\right)$is a function tabulated in Brons and Marting (1961).

Bradbury and Rothschild (1985) used regression to develop the following functional form from the tabulated values of G:

$$G\left(\frac{l}{b}\right) \cong 2.948 - 7.363\left(\frac{l}{b}\right) + 11.447\left(\frac{l}{b}\right)^2 - 4.675\left(\frac{l}{b}\right)^3 \tag{13}$$

The values tabulated by Brons and Marting (1961) are plotted in Figure 8 along with the regression of Bradbury and Rothschild (1985). As shown in the figure, the results obtained with the regression relation match closely the values in Brons and Marting (1961).

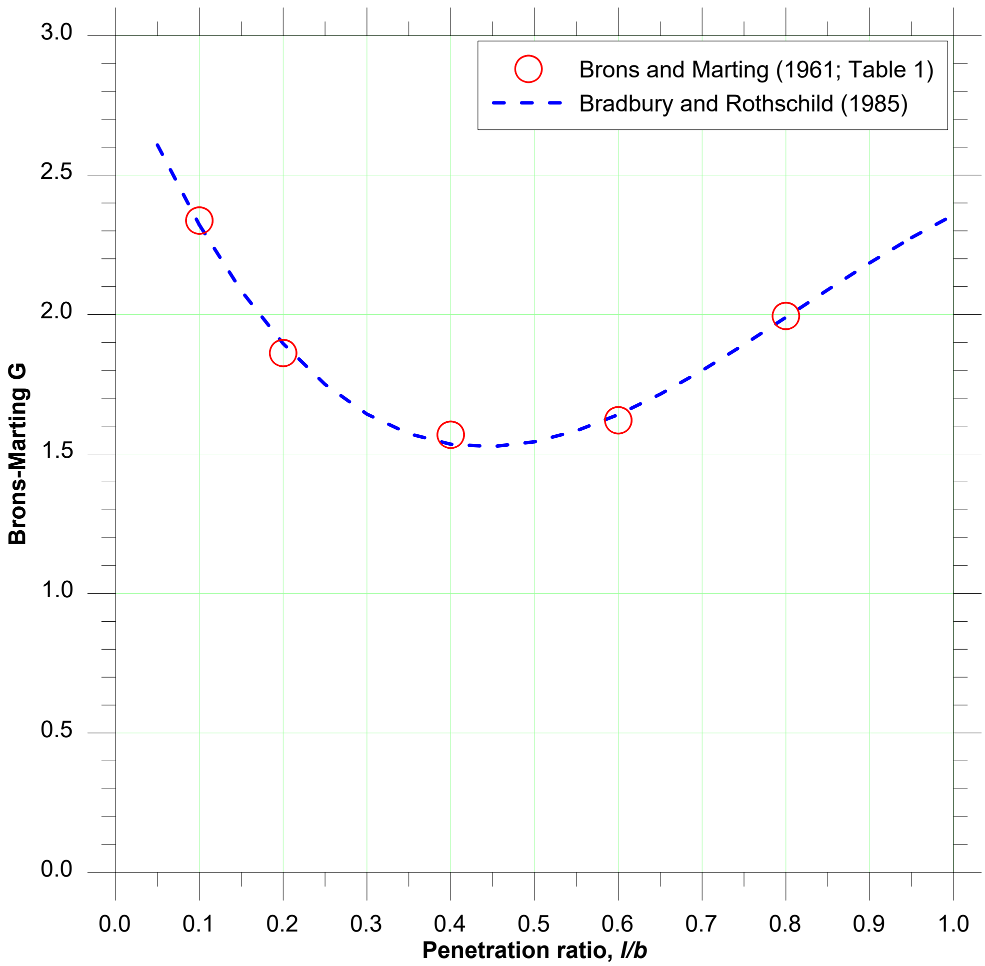


Figure 8. Values of the Brons-Marting function *G* for partially penetrating wells

6. Representation of the additional head losses due to turbulent flow near or within the well

The velocity of water in the immediate vicinity of the well may be sufficiently high that flow is turbulent. Flow may also be turbulent within the well casing itself and around the appurtenances may be relatively high, and flow may be turbulent. Jacob (1946) proposed a simple phenomenological approach for estimating the head losses due to turbulence in the well itself. There are two key aspects of the Jacob model of in-well turbulent losses:

- Turbulent losses are established relatively quickly after pumping starts; and
- Turbulent losses are proportional to the pumping rate squared.

The Jacob model is expressed as:

$$\Delta s_{turbulence} = CQ^2 \tag{14}$$

The parameter *C* is designated the *well loss coefficient*. Rorabaugh (1953) suggested that Jacob's model was not always appropriate, and proposed the following generalization:

$$\Delta s_{turbulence} = CQ^P \tag{15}$$

Here *P* is designated the *well loss exponent*. Rorabaugh reported exponents that were not too different from 2.0, and we recommend using Jacob's model except where there is compelling evidence that *P* should not be 2.0. The well loss coefficient *C* has the units of [drawdown]/[Pumping rate]^P. If the pumping rate is reported in m^3/day the units of drawdown are m, and it is assumed that $P = 2$, then *C* has units of $m/(m^3/day)^2$, which is equivalent to day^2/m^5.

The well loss coefficient *C* is a fitting parameter. The most reliable estimates of *C* are derived from the results of step tests, as discussed later in these notes. In the absence of site-specific data, we recommend that the general guidance provided by Walton (1962; p. 27) be used to assign preliminary values.

First-cut values of *C*

Condition of well	C (sec^2/ft^5)	C (day^2/ft^5)	C (sec^2/m^5)
Properly designed and developed	$C < 5$	$C < 6.7\times10^{-10}$	$C < 1900$
Mild deterioration	$C < 10$	$C < 1.3\times10^{-9}$	$C < 3800$
Well beyond rehabilitation	$C > 10$	$C > 1.3\times10^{-9}$	$C > 3800$

7. Preliminary estimation of the transmissivity from the specific capacity of a pumping well

Transmissivity data are frequently limited in regional groundwater studies. Controlled pumping tests with observation wells are often available at only a few locations. However, the drilling logs for domestic supply wells contain information that can supplement the available data. In particular, these logs generally report pumping data that can be used to calculate specific capacities for the wells, and these specific capacities can be correlated to transmissivity with simple models. These correlations yield reconnaissance-level estimates of transmissivity. Where more detailed data are available, specific capacity values can also serve to provide simple check on the interpretations. Here we describe a simple approach for estimating the transmissivity from specific capacity data. The crucial assumption of the analysis is that the drawdowns in the pumping well are due primarily to head losses in the formation.

The specific capacity is defined as the ratio of the pumping rate (Q) and the drawdown in the pumping well (s_w):

$$SC = \frac{Q}{s_w} \tag{16}$$

If well losses and any effects of wellbore storage are neglected, the specific capacity can be estimated by evaluating the Theis solution at the radius of the wellbore, r_w:

$$SC = \frac{Q}{s_w} = \frac{4\pi T}{W\left(\frac{r_w^2 S}{4Tt}\right)} \tag{17}$$

The transmissivity can be back-calculated from the reported value of the specific capacity with known or assumed values for the well radius and storage coefficient:

$$T = \frac{1}{4\pi} W\left(\frac{r_w^2 S}{4Tt}\right) \times SC \tag{18}$$

Equation (18) is an implicit function of the transmissivity T. Although it is possible to estimate T using a root-finding algorithm, a simpler approach is illustrated here. For a particular well size and duration of pumping, it is possible to use Equation (18) directly to plot the relation between the *SC* and *T*. The transmissivity can then be estimated directly from the plot. We can develop our own plots for typical well diameters and durations of pumping.

The relation between specific capacity and transmissivity for typical conditions reported in water well records in Ontario is shown in Figure 9. The relationship is shown for a typical range of storage coefficients for confined conditions ($S = 1\times10^{-5}$ to 1×10^{-3}). The results plotted in Figure 9 demonstrate that the specific capacity is relatively insensitive to the value assumed for the storage coefficient.

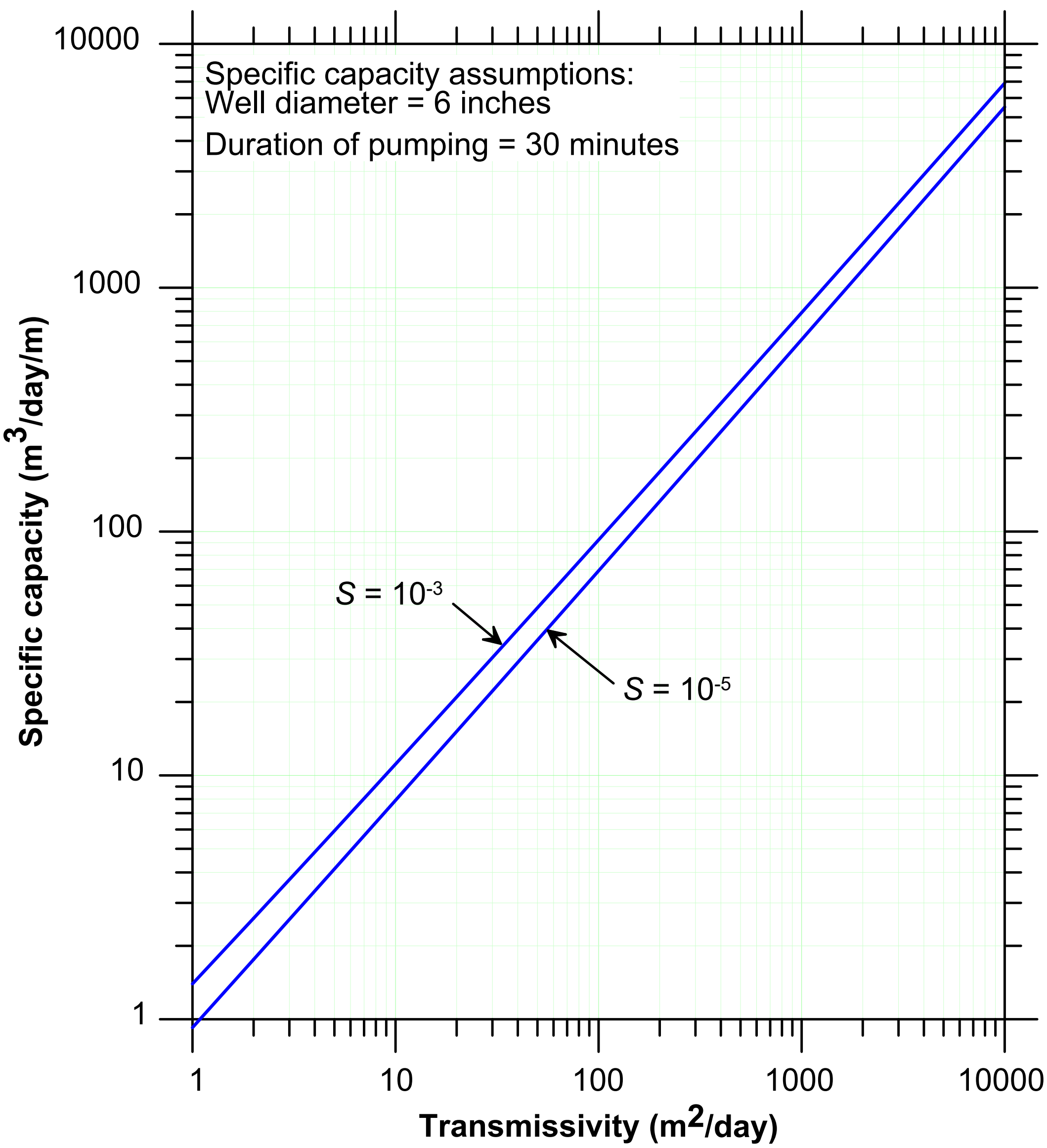


Figure 9. Specific capacity-transmissivity relation

The results shown in Figure 9 suggest that the log-transformed specific capacity is nearly a linear function of the log-transformed transmissivity range of 1 to 10,000 m^2/day. As shown in Figure 10, over this range the exact results are matched relatively closely with the simple relation (assuming consistent units):

$$T \approx 1.3 \times SC \tag{19}$$

The simplified relation is superimposed on the exact results in Figure 10. The good match between Equation (19) and the exact results suggests that Equation (19) is appropriate for developing reconnaissance-level estimates of transmissivity.

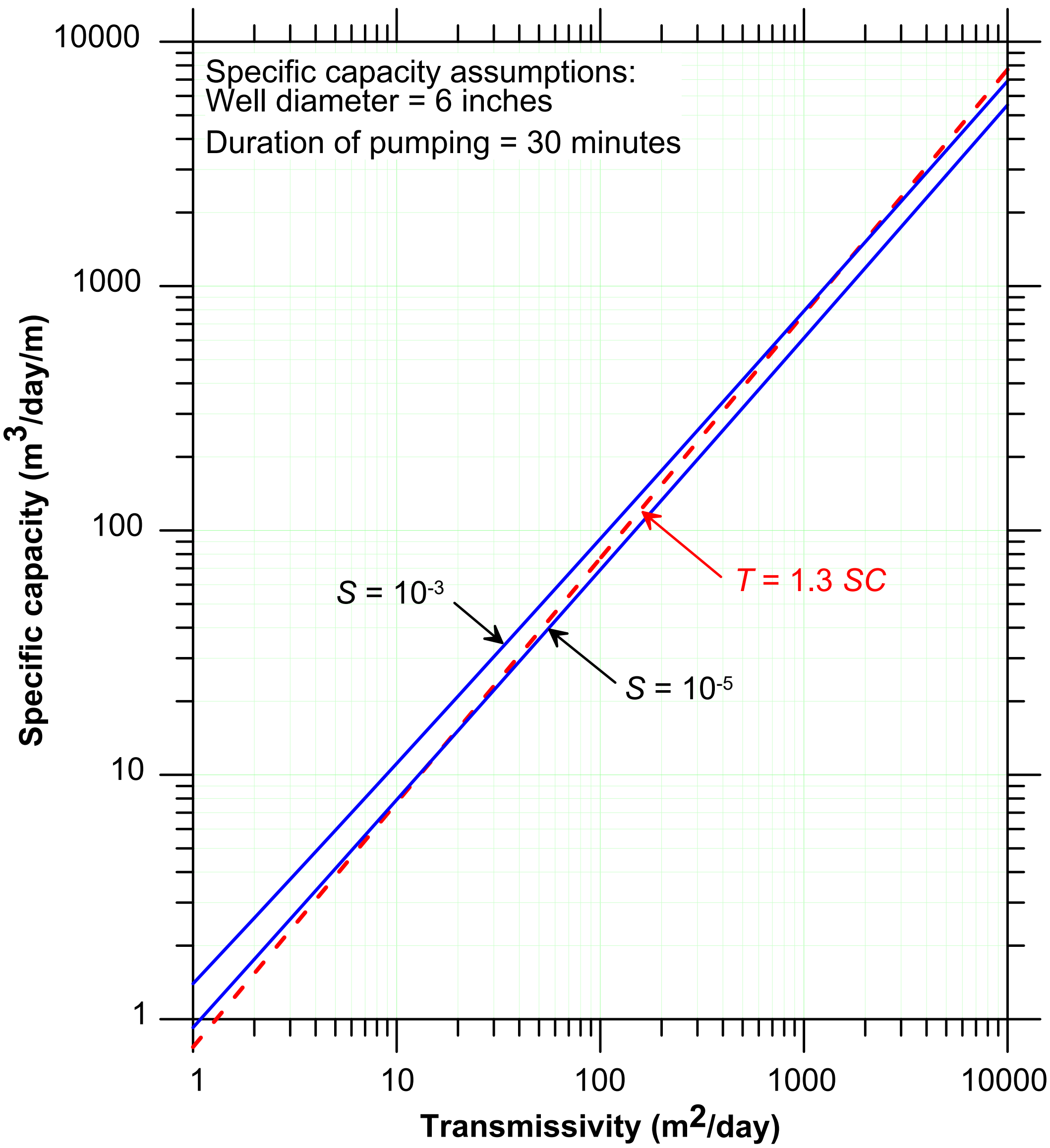


Figure 10. Specific capacity-transmissivity relation, with suggested correlation

If the specific capacity is specified in terms of U.S. gallons per minute (gpm) per foot of drawdown, and the transmissivity is reported in units of gallons/day-ft, the correlation becomes:

$$T \approx 1750 \times SC \tag{20}$$

The leading coefficient of 1750 is close to the value of 2000 presented in Driscoll (1986, p. 1021), which assumes the well is pumped for 1 day. The inferred correlation is superimposed on results plotted in Walton (1970, p. 317) in Figure 11 for a pumping period of 10 minutes. The results match closely, suggesting that the inferred correlation is appropriate for the shorter pumping periods typically reported in the water well records.

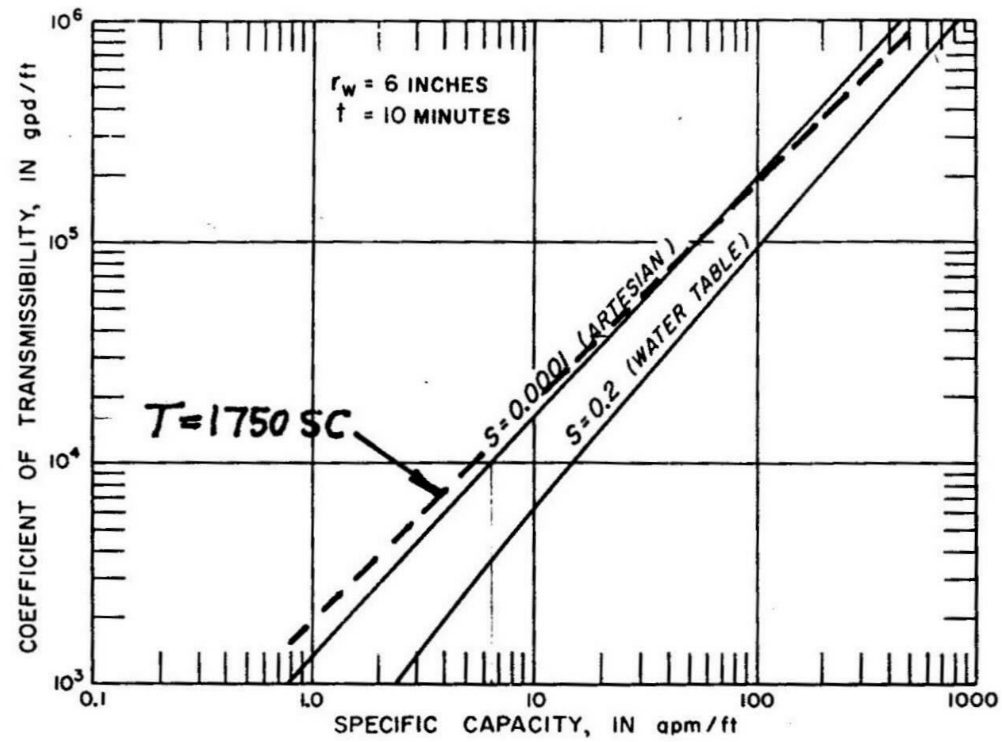


Figure 11. Specific capacity-transmissivity relation for brief pumping

The specific capacity is a weak function of the well radius and duration of pumping. The results of additional calculations suggest that the correlation equations (19) and (20) do not need to be modified significantly to accommodate different well sizes or durations of pumping.

Case study: Rosemont, Ontario well PW3

The first-cut estimation of transmissivity from the specific capacity is also useful for conducting a quick check on more complete analyses. The data from a pumping test conducted at Rosemont, Ontario is used to illustrate the approach. Well PW3 was pumped for three days at an average rate of 0.6 L/s (51.84 m^3/d). The complete record of drawdowns is shown in Figure 12. The drawdown at the end of 60 minutes of pumping is 5.94 m. Therefore, the specific capacity after 60 minutes is:

$$SC = \frac{(51.84\ \mathrm{m^3/d})}{(5.94\ \mathrm{m})} = 8.73\ \mathrm{m^3/d/m}$$

The transmissivity estimated from specific capacity is:

$$T \approx 1.30 \times (8.73\ \mathrm{m^3/d/m}) = \mathbf{11.3\ m^2/d}$$

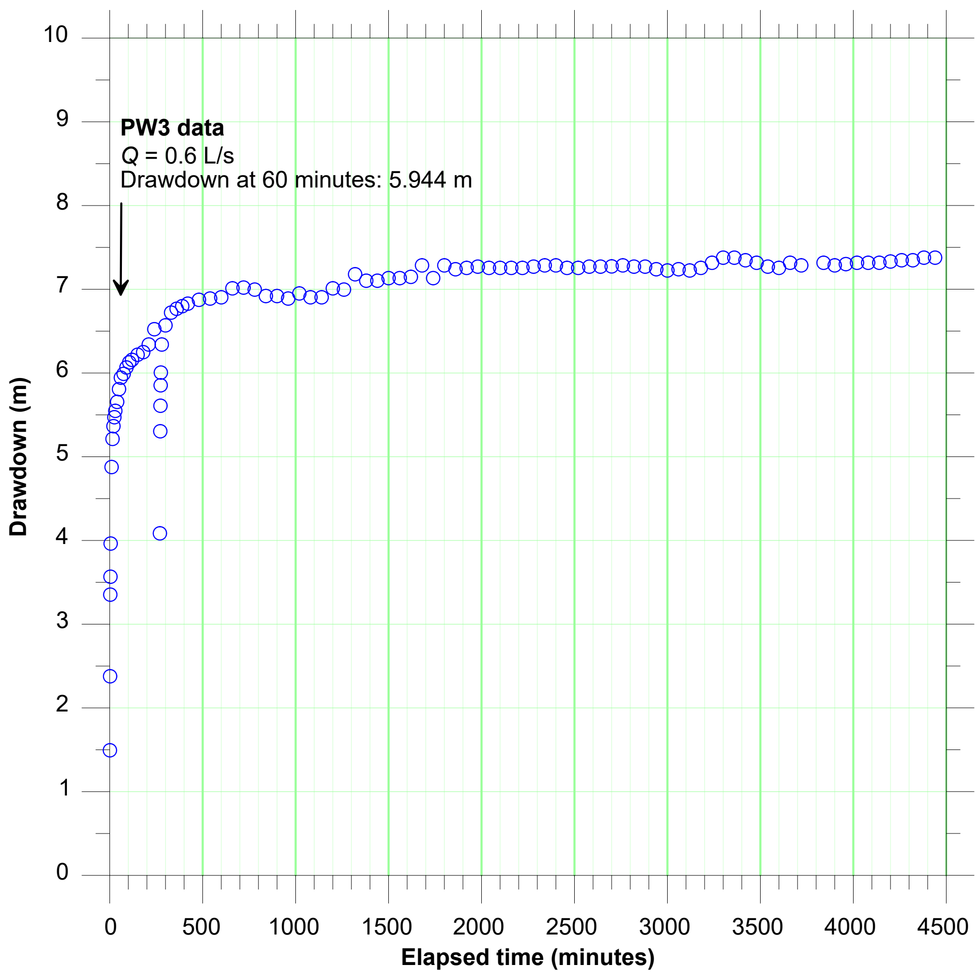


Figure 12. Drawdown record for the PW3 pumping test

The results of more rigorous analyses are shown in Figure . The transmissivity is estimated with the Cooper-Jacob analysis and with a match to the complete drawdown record with the Papadopulos and Cooper (1967) solution. A transmissivity of about **11 m^2/day** is estimated from both analyses. The close agreement between the two analyses suggests that well losses do not have a significant influence on the estimation of transmissivity for this test.

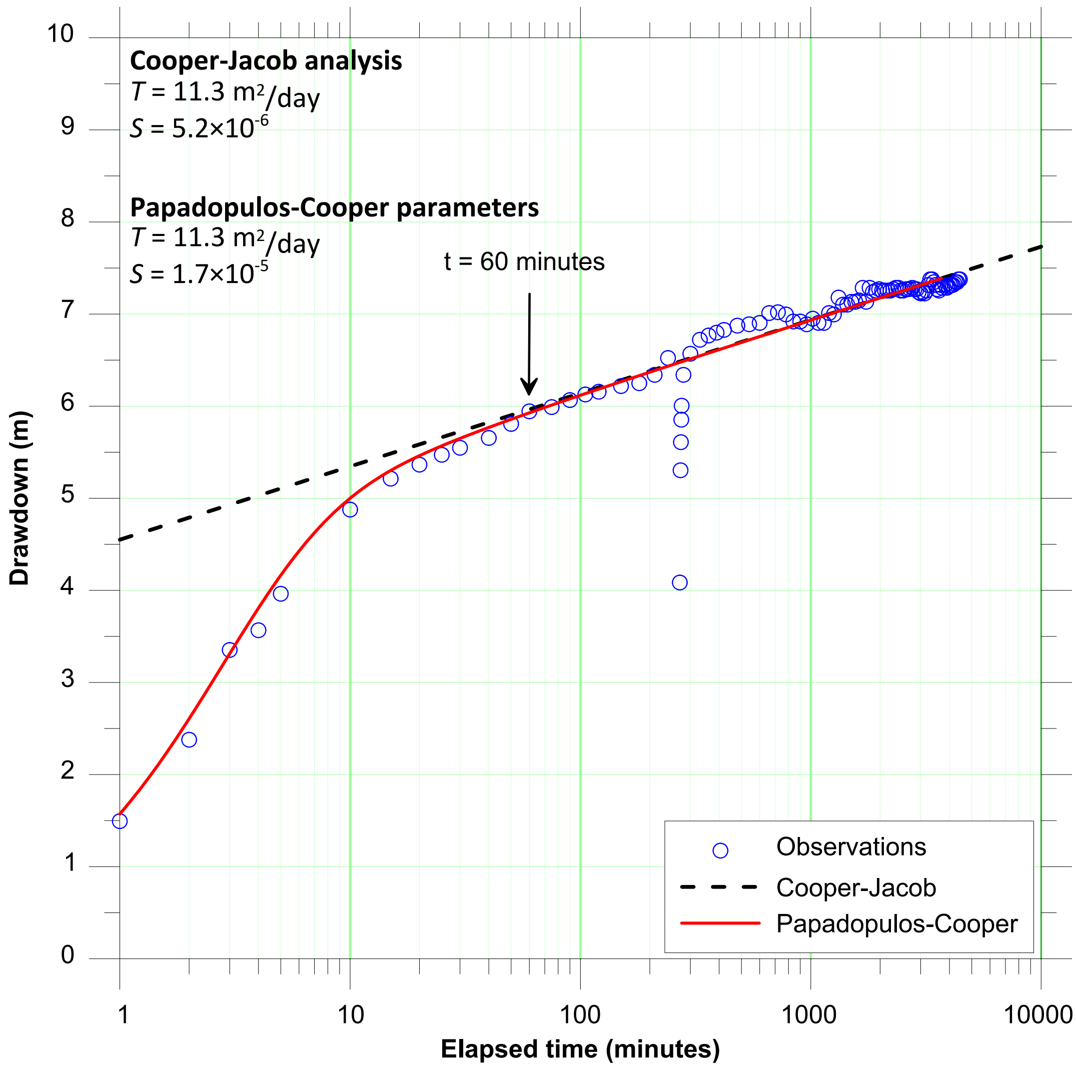


Figure 13. Rigorous analyses of the PW3 pumping test

The transmissivity estimated from the specific capacity for the Roseville well is close to the estimates developed from the more rigorous analyses of the complete drawdown record. This is not simply fortuitous. The availability of a complete drawdown record allows us the opportunity to confirm the following *in this case*.

- The time corresponding to the drawdown specified in the calculation of the specific capacity was sufficiently long for the effects of wellbore to dissipate. Referring to Figure , after about 30 minutes of pumping the differences between the Papadopulos and Cooper (1967) solution and the Cooper-Jacob straight line approximation are relatively small. This suggests that the effects of wellbore storage are almost completely dissipated within about 30 minutes so that the drawdown measured at 60 minutes provides a representative impression of the response of the formation.

- The early-time drawdowns are relatively small, which suggests that additional well losses are not significant. Therefore, the observed drawdowns in the pumping well provide a reliable impression of the head losses in the formation in the vicinity of the pumping well.

- The storage coefficients estimated from the Papadopulos-Cooper and Cooper-Jacob analyses are within the range of realistic storage coefficients for a confined aquifer that is relatively thin. This is consistent with the inference that the primary component of the observed drawdowns is head losses within the formation.

8. Complete transient analysis with the incorporation of additional well losses

Drawdown data from typical tests confirm that the additional drawdowns due to skin effects and in-well losses are established relatively soon after pumping starts, compared with the head losses in the formation. If it is assumed that the Theis conceptual model is applicable, substituting the expressions introduced previously for Δs_{skin} and $\Delta s_{turbulence}$ in Equation (1) yields the following expression for the evolution of the total drawdown in a pumping well:

$$s_w(t) = \frac{Q}{4\pi T}\, 2.303 \log\left\{2.246\,\frac{Tt}{r_w^2 S}\right\} + \frac{Q}{4\pi T}\, 2S_w + CQ^2 \qquad (21)$$

Here it is assumed that effects of wellbore storage have dissipated. When evaluated at small values of the radial distance *r*, the Cooper-Jacob approximation is appropriate for all but the earliest values of time.

Expanding the log term:

$$s_w(t) = \frac{Q}{4\pi T}\, 2.303\left(\log\left\{2.246\,\frac{T}{r_w^2 S}\right\} + \log\{t\}\right) + \frac{Q}{4\pi T}\, 2S_w + CQ^2$$

Re-arranging:

$$s_w(t) = \frac{Q}{4\pi T}\, 2.303\ \log\{t\} + \frac{Q}{4\pi T}\, 2.303 \log\left\{2.246\,\frac{T}{r_w^2 S}\right\} + \frac{Q}{4\pi T}\, 2S_w + CQ^2 \qquad (22)$$

The first term is a function of time, but the other three terms are constant. In other words, the time rate of change of drawdown is not affected by the processes that cause additional head losses in the pumping well. Since the Cooper-Jacob analysis is based on the rate of change of drawdown rather than the absolute magnitude of the drawdown, it is possible to obtain a reliable estimate of the transmissivity from a Cooper-Jacob straight-line analysis, regardless of the magnitudes of the skin losses and turbulent well losses.

Example calculations:

Let us consider an ideal aquifer that is homogeneous, horizontal, perfectly confined, infinite in extent, and pumped by a fully penetrating well. The aquifer is assumed to have a transmissivity of 8.64 m^2/day and a storativity of 1.0×10^{-4}. These properties are typical of a medium sand aquifer that is 10 m thick. The aquifer is pumped at a constant rate of 104.54 m^3/day, and the pumping well has a radius of 0.05 m. Let us further assume that the pumping well losses are characterized by the following parameters:

- $S_w = 0.5193$; and
- $C = 1.340\times10^{-4}\ \mathrm{m^{-5}d^2}$.

These values have been specified only for illustrative purposes – under no circumstances would we report estimates from a real test with so many significant figures.

The total drawdowns in the pumping well are plotted in Figure 14. The dashed line in Figure 14 indicates the drawdowns in the pumped well that are due only to head losses in the formation. The drawdown axis is arithmetic, and therefore the additional head losses in the pumping well appear as a constant offset. Both drawdown curves have the same slope on a Cooper-Jacob semilog plot.

The slope of the line plotted through the drawdown data is approximately 2.2 m. Therefore, the transmissivity is estimated as:

$$T = 2.303\frac{Q}{4\pi}\frac{1}{SLOPE}$$

$$= 2.303\frac{(104.54\ \mathrm{m^3/d})}{4\pi}\frac{1}{(2.2\ \mathrm{m})} = \mathbf{8.7\ m^2/d}$$

The estimated transmissivity is close to the specified value of 8.64 m^2/d.

The transmissivity with the Cooper-Jacob analysis is estimated from the semilog slope of the drawdowns and *not* the magnitudes of the drawdowns. Therefore, for a constant-rate pumping test the transmissivity estimate is not affected by the constant offsets of the additional well losses.

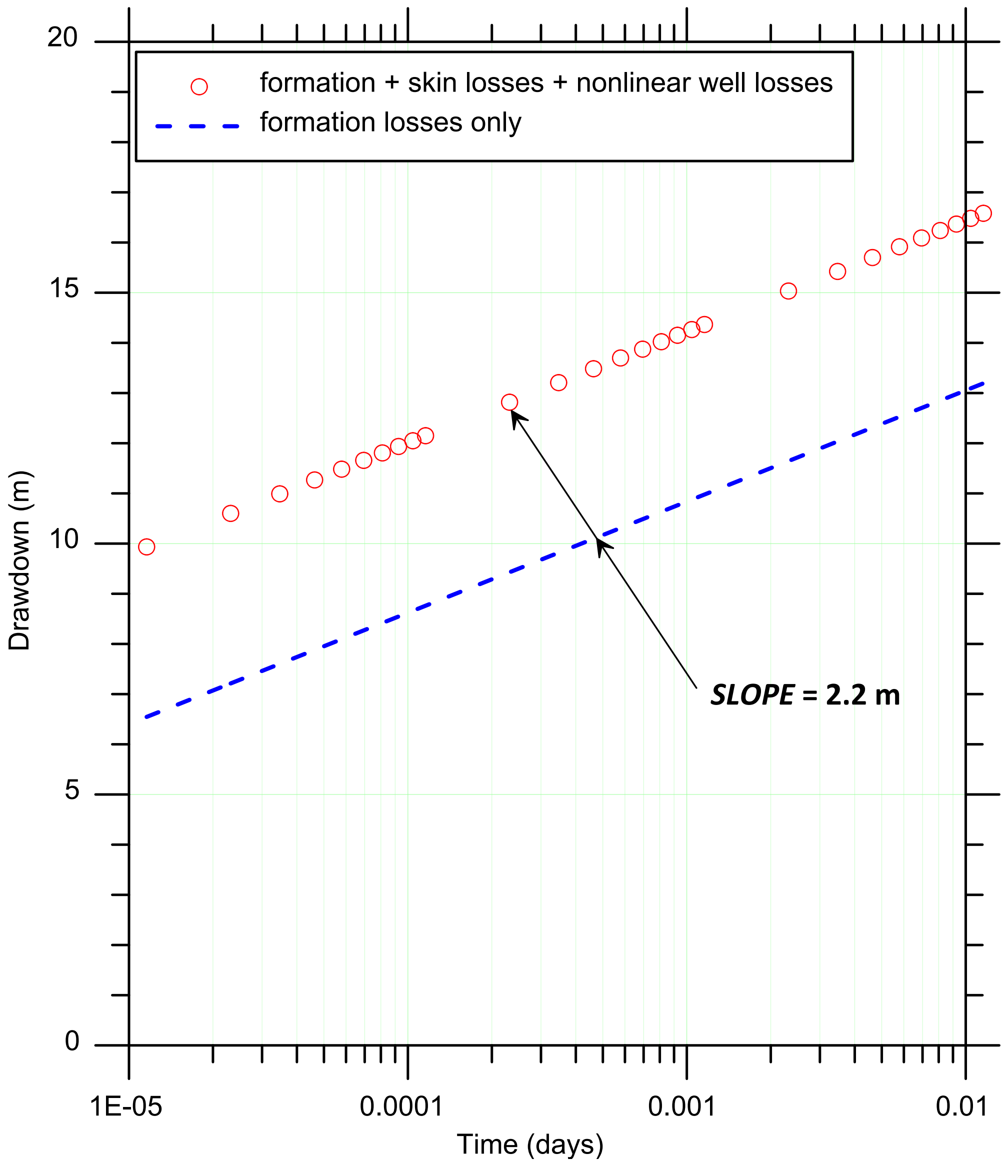


Figure 14. Drawdowns at the pumping well

9. Diagnosis of additional well losses

In the context of the analysis of the drawdowns in a pumping well, the estimation of a non-physical value of the storage coefficient is frequently a good indicator of the presence of additional well losses.

The storage coefficient is estimated with the Cooper-Jacob analysis according to:

$$S = 2.2459 \frac{T t_0}{r_w^2}$$

Here t_0 is the intercept of the straight-line approximation. Referring to the expanded version of Figure 14 shown in Figure 15, the value of t_0 is about 4×10^{-10} days. Therefore, the fitted storage coefficient is estimated as:

$$S = 2.2459 \frac{(8.64 \text{ m}^2/\text{d})(4 \times 10^{-10} \text{ days})}{(0.1 \text{ m})^2} = \mathbf{7.7 \times 10^{-7}}$$

The "fitted" storage coefficient is more than a factor 100 less than the specified value and is well outside of the range of typical values of the storage coefficient for confined sand and gravel aquifers, from about 10^{-5} to 10^{-4}.

The storage coefficient is estimated from the intercept of the plot; therefore, in contrast to the estimation of the transmissivity, the inferred magnitude of the storage coefficient *does* depend on the magnitudes of the drawdowns. When we use the Cooper-Jacob straight-line analysis, we effectively estimate a storativity that accounts in a "lumped" sense for the effects of storage and additional well losses. Although we do not obtain a true estimate of the storage coefficient, its estimation still has useful diagnostic value. Estimation of an unrealistic value of the storage coefficient suggests there are additional sources of drawdown beyond head losses in the formation.

If all we have are the data from the pumping well when it is pumped at a constant rate, then we must accept the fact that we cannot obtain reliable estimates of the storativity and the well loss parameters. The data are not sufficient to characterize the performance of the pumping well.

We use this example to illustrate another subtle point. In Figure 16 the drawdowns in the pumping well are re-plotted on log-log axes in anticipation of matching the observations with the Theis solution. In contrast to the Cooper-Jacob analysis, the additional head losses in the pumping well do not appear as a constant offset. They are in fact difficult to detect on a log-log plot. Furthermore, the estimate of transmissivity will be affected by the additional well losses.

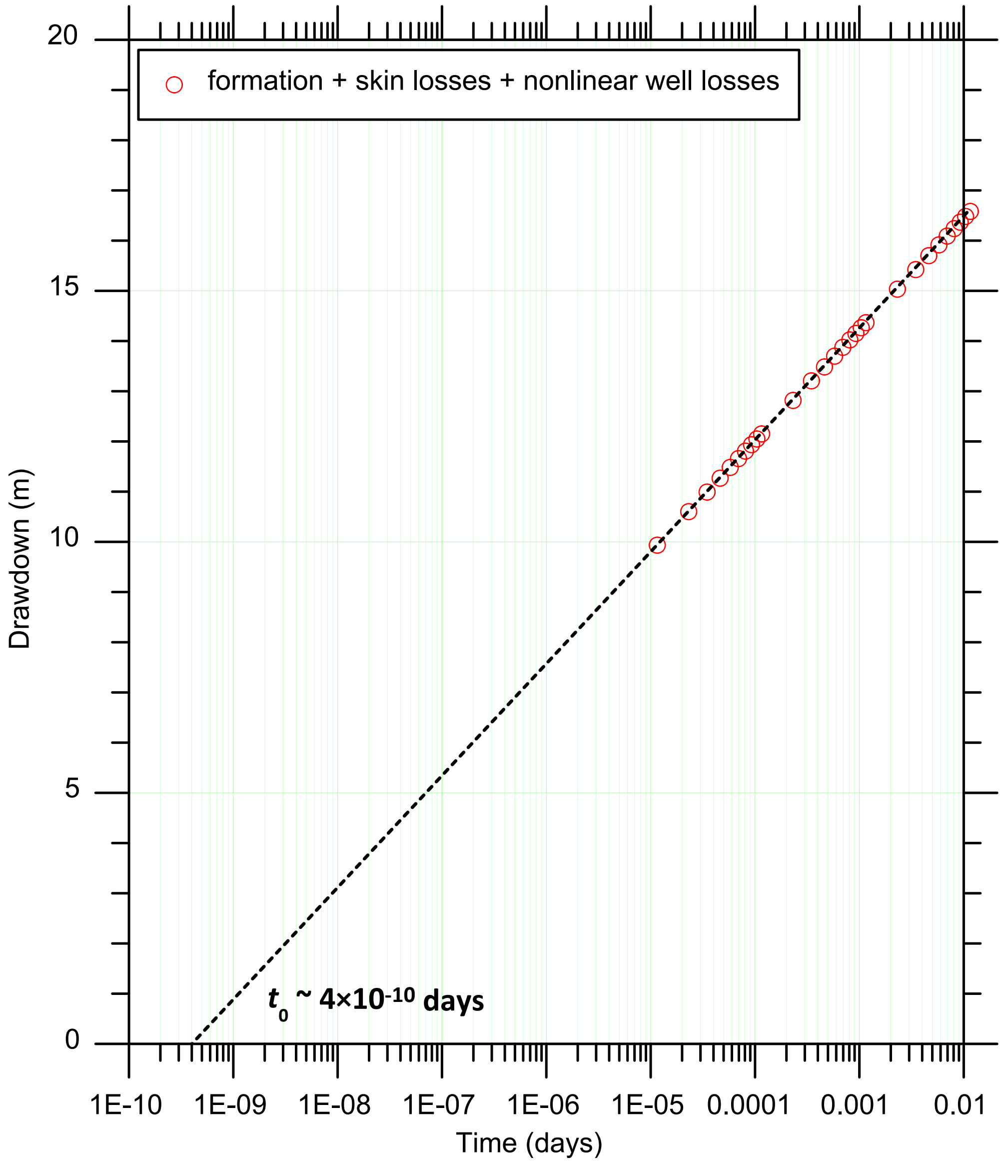


Figure 15. Drawdowns at the pumping well, with expanded time axis

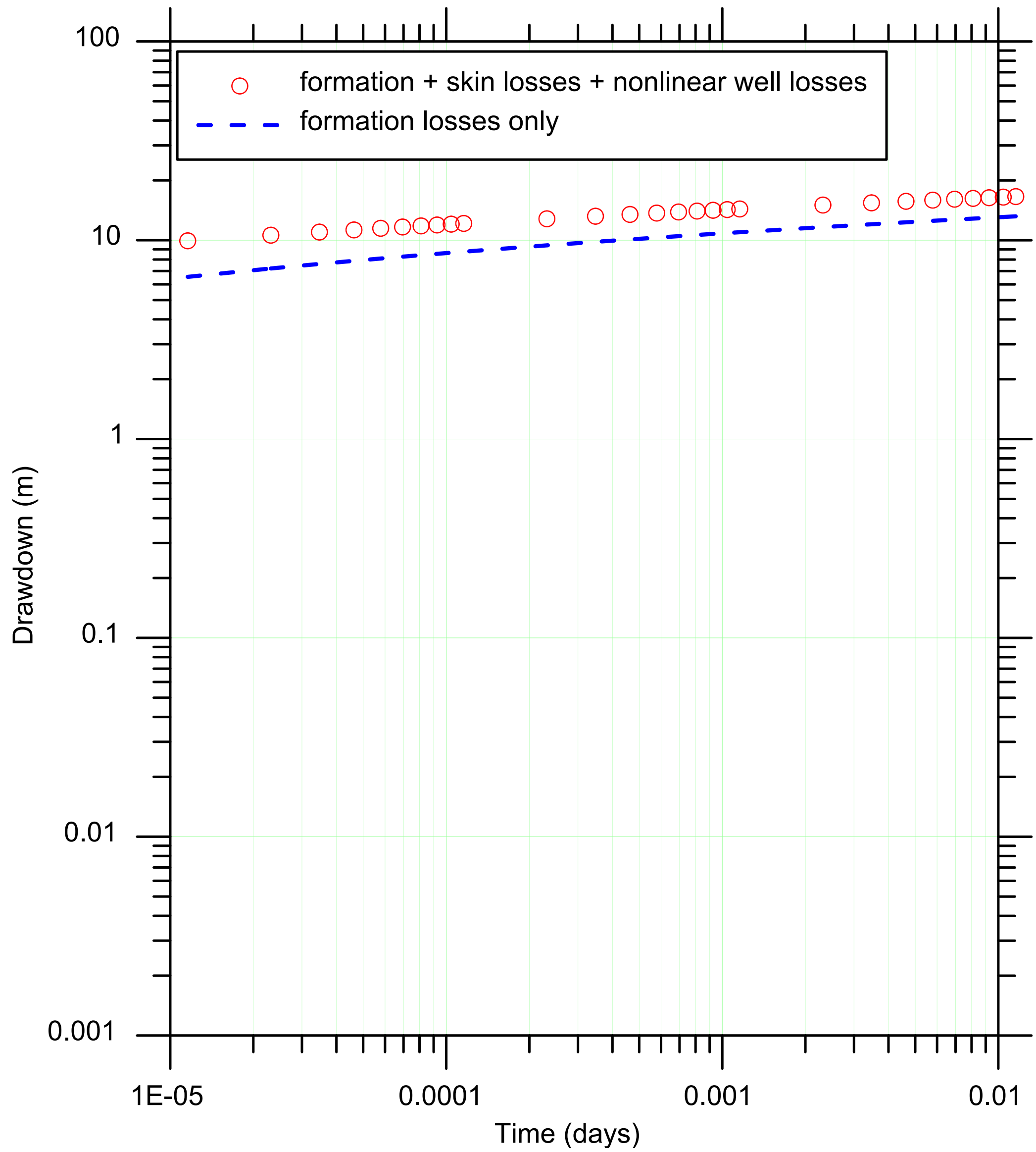


Figure 16. Drawdowns at the pumping well, log-log axes

10. Further investigation of additional well losses

As indicated in the previous section, the estimation of a non-physical value of the storage coefficient from the pumping well drawdowns during a constant-rate test is frequently a good indicator of the presence of additional well losses. This is explored further for a simple example of a well surrounded by a skin zone.

Assuming that the drawdowns in a pumping well are attributed only to laminar head losses in the formation and head losses across the skin, the drawdown in the well is given by:

$$s_w(t) = \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{{r_w}^2 S}{4Tt}\right\}\right] + \frac{Q}{4\pi T} 2S_w \tag{23}$$

Equation (23) can be expanded as:

$$s_w(t) = \frac{Q}{4\pi T}\left[ln\{EXP\{-0.5772\}\} - ln\left\{\frac{{r_w}^2 S}{4Tt}\right\} + ln\{EXP\{2S_w\}\}\right]$$

Making use of the properties of the log function:

$$s_w(t) = \frac{Q}{4\pi T} ln\left\{EXP\{-0.5772\}\frac{4Tt}{{r_w}^2 S}\ EXP\{2S_w\}\right\}$$

$$= \frac{Q}{4\pi T} ln\left\{\frac{2.246\,Tt}{{r_w}^2 S\ EXP\{-2S_w\}}\right\}$$

Changing to base 10 logarithms:

$$s_w(t) = \frac{Q}{4\pi T} 2.303\, log_{10}\left\{\frac{2.246\,Tt}{{r_w}^2 S\ EXP\{-2S_w\}}\right\} \tag{24}$$

In this form we see that the pumping well drawdowns correspond to those that would be matched with the Cooper-Jacob approximation with an “effective” storage coefficient given by:

$$S_{eff} = S\ EXP\{-2S_w\} \tag{25}$$

The results of example calculations are shown in Figure 17. For these calculations a typical value of the storage coefficient for a confined sand and gravel aquifer is assumed, $S = 1.0\times10^{-4}$. As shown in the figure, when the pumping well is surrounded by a zone of reduced hydraulic conductivity (positive skin), an unrealistically small value of the storage coefficient is inferred from a Cooper-Jacob analysis ($S_{eff} \sim 2.0\times10^{-6}$). In contrast, when the pumping well is surrounded by a zone of increased hydraulic conductivity (negative skin), an unrealistically large value of the storage coefficient is inferred from a Cooper-Jacob analysis ($S_{eff} \sim 5.0\times10^{-3}$).

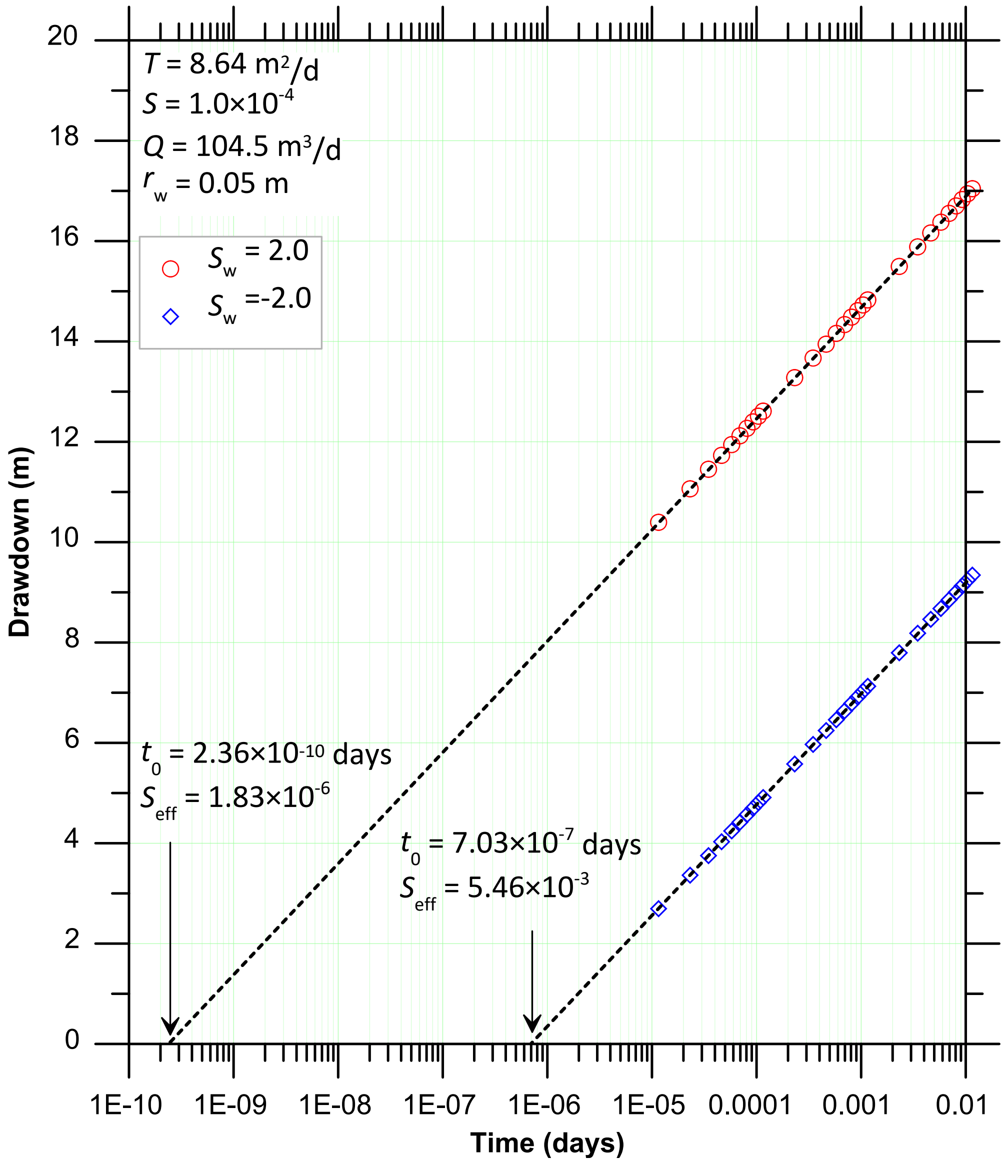


Figure 17. Example pumping well drawdowns with skin zones

11. Step tests

If the only data that are available are from the pumping well, and we want to obtain reliable estimates of the transmissivity, storativity, <u>and</u> well loss characteristics, we must monitor the level in the well as it is pumped at different rates. Each interval of pumping at a constant rate is referred to as a step, and this testing sequence is referred to as a *step test* (Jacob, 1946). An example dataset is shown in Figure 18.

There are two general approaches for interpreting the results of step tests:

- Steady-state analysis: the pumping well drawdowns are interpreted as if they were obtained from a sequence of steady-states; and
- Transient analysis: The entire time history of drawdowns is analyzed.

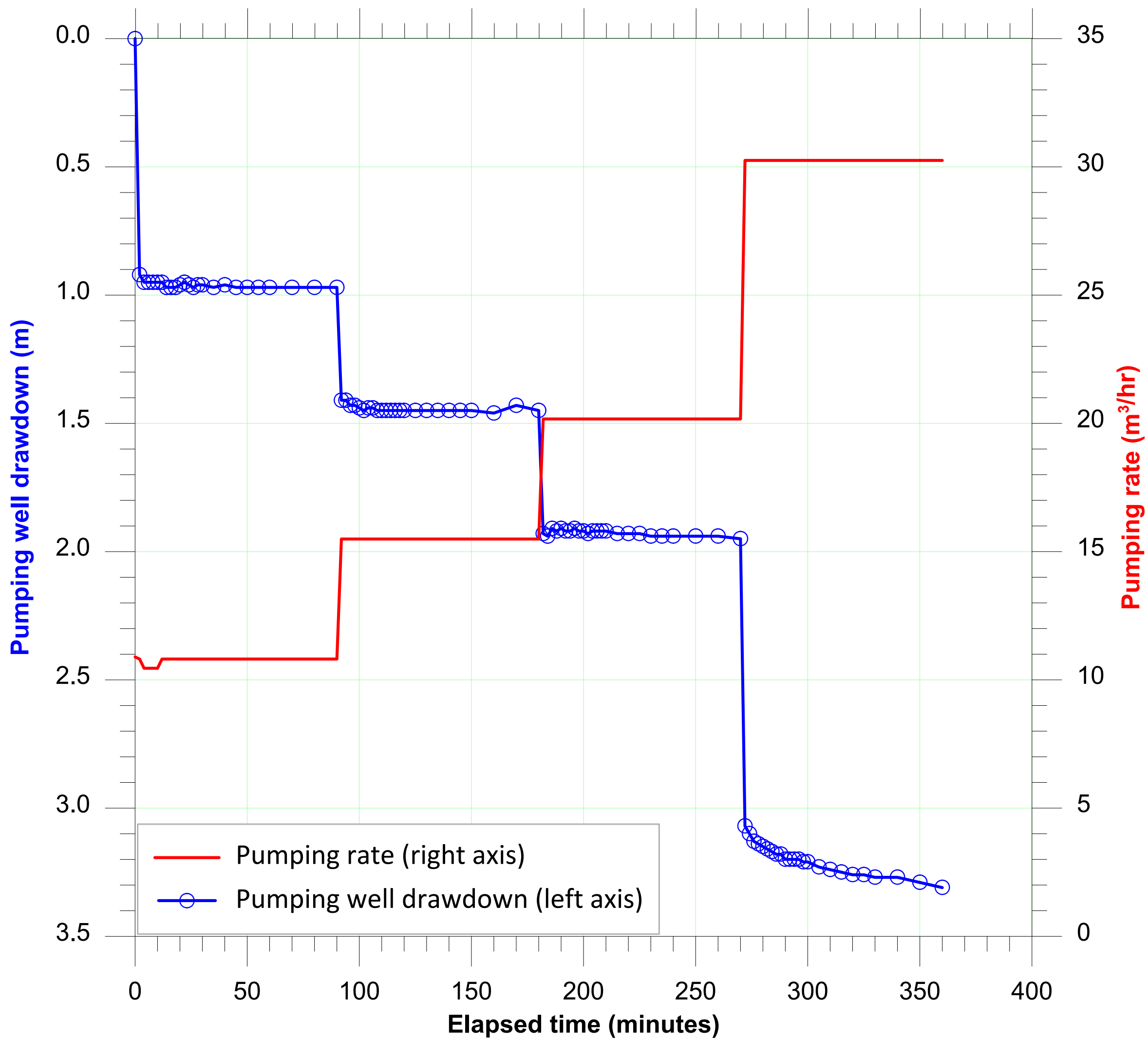


Figure 18. Example step test data

12. Step test interpretation: Steady-state analysis

If stabilized drawdowns in the pumping well are available for several different pumping rates, a particularly simple method is available to diagnose turbulent well losses and estimate the well loss coefficient *C*.

Since the head losses due to flow in the formation and flow across the skin zone are linear functions of the pumping rate, the total drawdown can be written as:

$$s_w = BQ + CQ^2 \tag{26}$$

Here B is a lumped parameter that accounts for head losses in the formation and additional head losses across the skin zone. Following the original work of Jacob (1946), it is assumed here that the well loss exponent P is 2.0. Dividing both sides of Equation (23) by *Q* yields:

$$\frac{s_w}{Q} = B + CQ \tag{27}$$

The quantity s_w/Q is referred to as the *specific drawdown*. Equation (27) predicts that if there are nonlinear well losses, the specific drawdown will increase linearly with the pumping rate.

An application of the steady-state approach is shown in Figure 19. The specific drawdown, s_w/Q is plotted against the pumping rate *Q* for each step. Figure 19 is referred to as a *Hantush-Bierschenk plot* after its initial developers Hantush (1964) and Bierschenk (1964). If the specific drawdowns approximate a straight line, we can infer:

- The slope of the line corresponds to the nonlinear well loss coefficient, *C*; and
- The intercept of the line, *B*, corresponds to the specific capacity of the well with the nonlinear well losses removed.

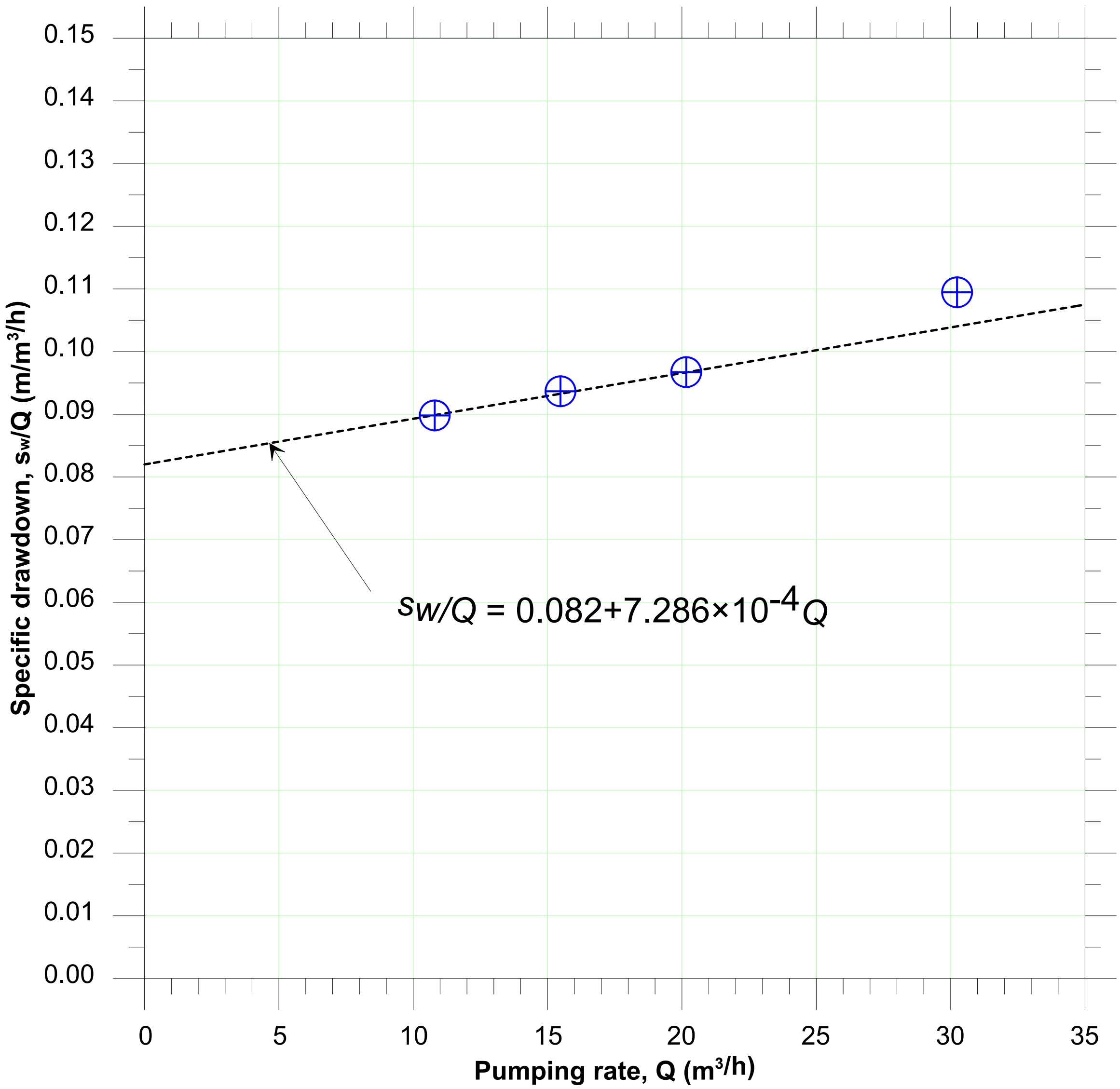


Figure 19. Hantush-Bierschenk plot, data from Figure 18

Case study: PW6/63, Guelph, Ontario – Part 1

Step tests were conducted in 1996-1998 on municipal supply wells in Guelph, Ontario, as part of a city-wide aquifer performance investigation (Jagger Hims Ltd., 1998). The discharge and drawdown data are of sufficient quality and frequency to support detailed analyses. Our analysis of the data for well PW6/63 follows a phased approach of increasing complexity:

- Diagnosis of nonlinear well losses;
- Estimation of transmissivity from the corrected specific capacity; and
- Estimation of transmissivity from a transient analysis.

The data from the step test are plotted in Figure 20.

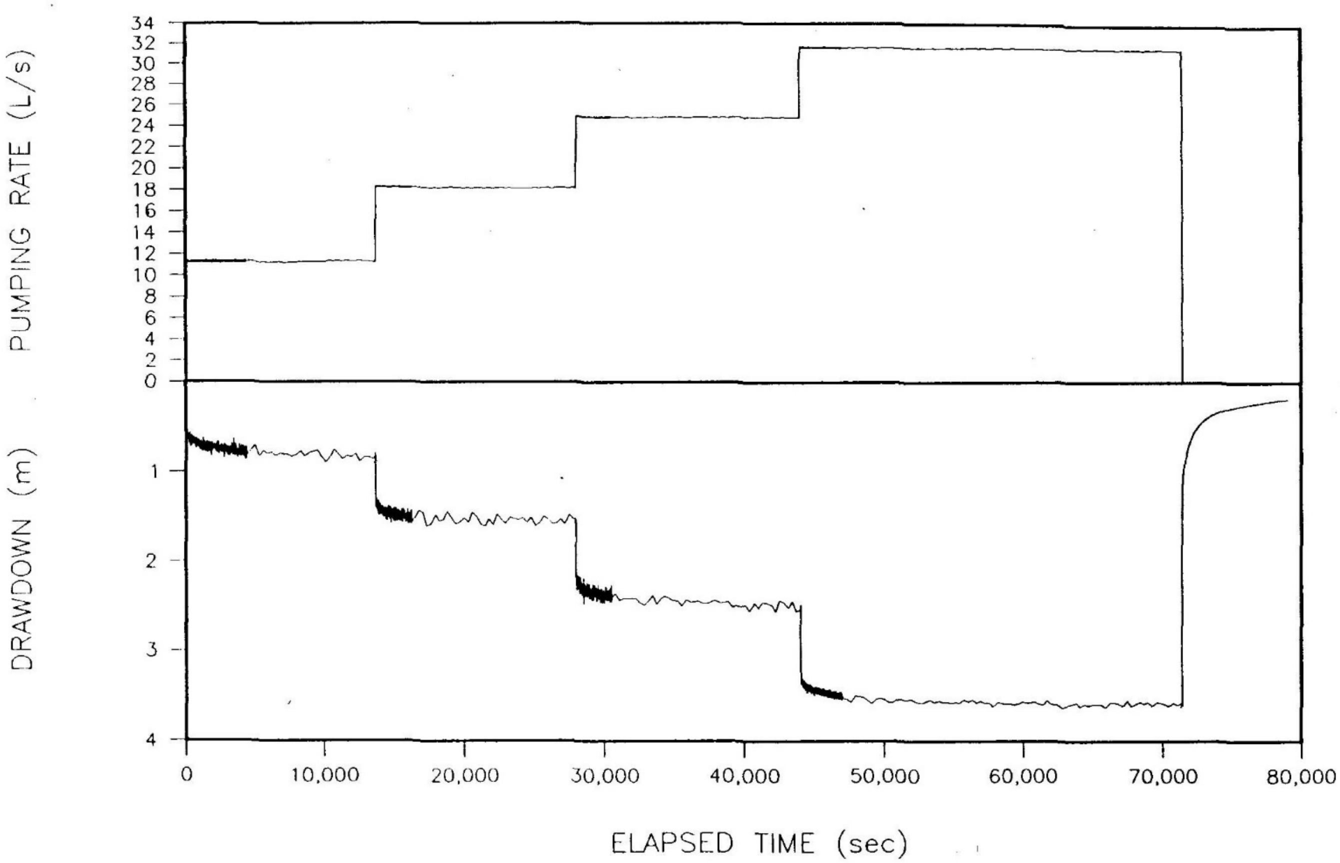


Figure 20. PW6/63 step test data

Step 1: Diagnosis of nonlinear well losses

The drawdowns appear to stabilize by the end of each pumping step. The pumping rates and drawdowns recorded at the end of each pumping step are tabulated below and plotted in Figure 21.

Step	Pumping rate, Q (L/s)	Drawdown, s_w (m)
1	11.5	0.87
2	18.6	1.53
3	25.3	2.46
4	32.0	3.60

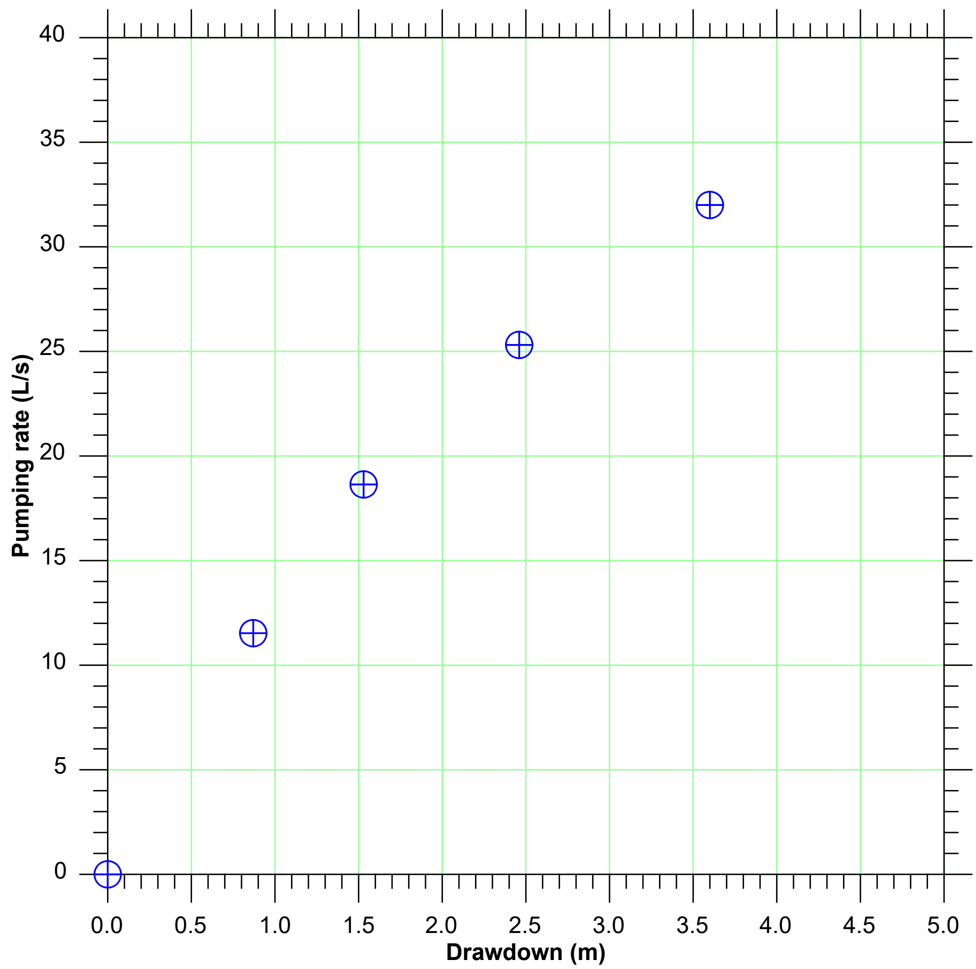


Figure 21. "Raw" specific capacity results

Under ideal conditions, that is, when all head losses are linear, the specific capacity estimated as the ratio of the pumping rate and drawdown at any particular pumping rate is identical to the slope of the relation between the pumping rate and the drawdown. However, as shown in Figure 21, during this test the specific capacity varies with the pumping rate. A reduction of the specific capacity for increased pumping rates is an initial suggestion that the pumping well drawdowns include nonlinear head losses.

To evaluate the nonlinear well losses, the results at the end of each step are assembled on a Hantush-Bierschenk plot in Figure 22.

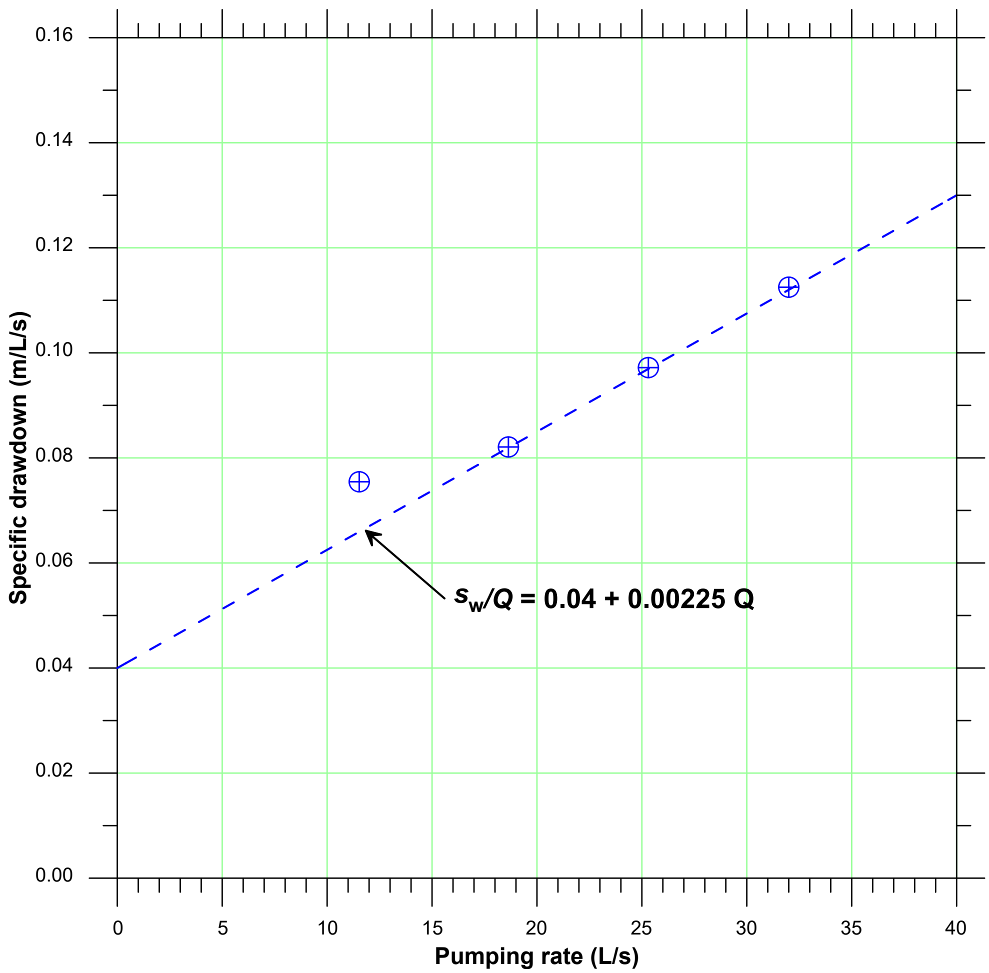


Figure 22. Specific drawdown vs. pumping rate

As shown in Figure 12, the relation between the specific drawdown and the pumping rate is nearly linear. This linearity of the plot of s_w/Q versus Q suggests that the pumping well drawdowns may be approximated using the Jacob (1946) model:

$$s_w = BQ + CQ^2$$

The parameters estimated with a linear regression analysis are:

- $\boldsymbol{B}$ **= 0.04 m/(L/s)**; and
- $\boldsymbol{C}$ **= 2.25×10^{-3} m/(L/s)^2**.

As a check, we use the fitted relation to plot the pumping rates as a function of the drawdown. As shown in Figure 23, the Jacob model matches the drawdowns closely.

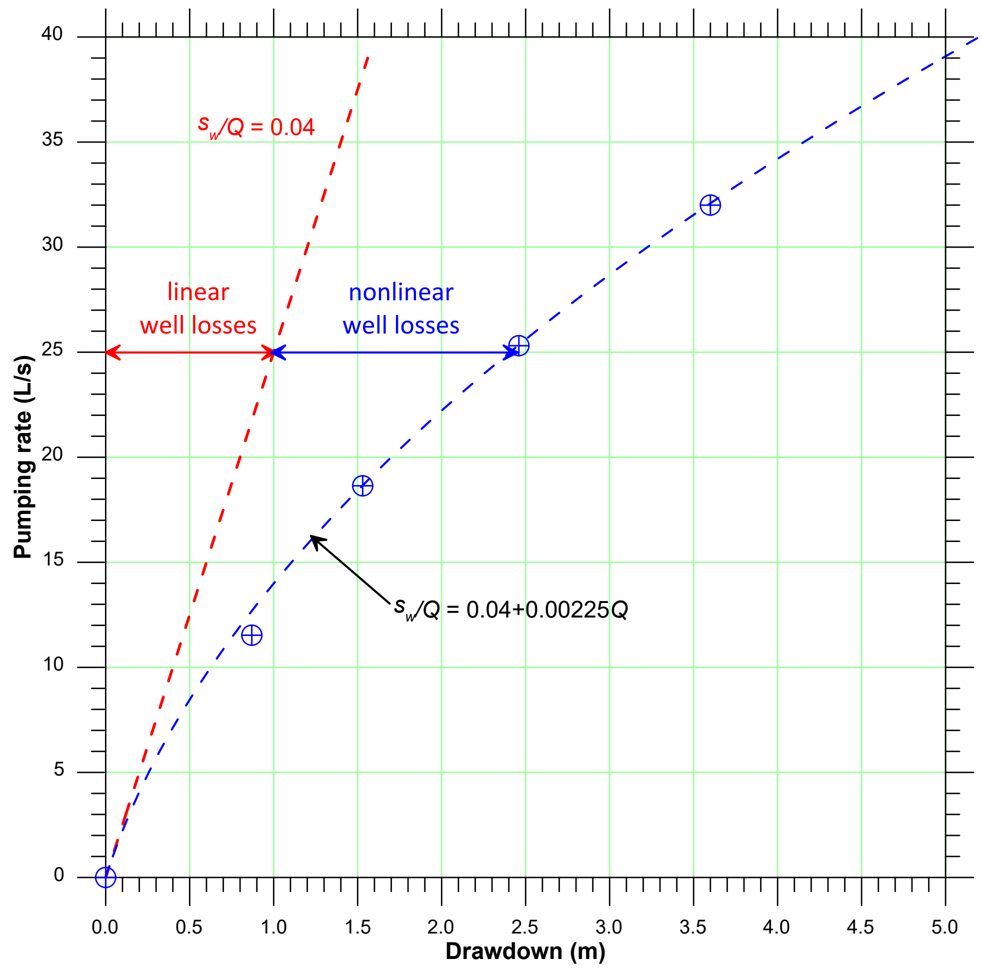


Figure 23. Specific capacity with inferred relation from the Hantush-Bierschenk plot

Step 2: Estimation of transmissivity from the "corrected" specific capacity

The coefficient B has an important physical interpretation. If we assume that skin losses are negligible, B represents the inverse of the specific capacity with the nonlinear well losses removed. A preliminary estimate of the transmissivity using the well-loss-removed specific capacity can be developed using a simple reconnaissance-level approach (Driscoll, 1986):

$$T \approx 1.4\, SC$$

The specific capacity with nonlinear well losses removed is given by:

$$SC = \frac{1}{B} = \frac{1}{0.04\ \mathrm{m/(L/s)}} = 25\ \mathrm{(L/s)/m}$$

Therefore, the transmissivity is estimated as:

$$T = 1.4\left(25\ \frac{\mathrm{L}}{\mathrm{s}}/\mathrm{m}\right) \approx 35\frac{\frac{\mathrm{L}}{\mathrm{s}}}{\mathrm{m}}\left|\frac{86{,}400\ \mathrm{s}}{\mathrm{d}}\right|\left|\frac{\mathrm{m}^3}{1000\ \mathrm{L}}\right| = \mathbf{3{,}020\ m^2/d}$$

13. Step test interpretation: Transient analysis

If a complete time history of drawdown is available during a step test, we can try to make use of all of the data in a transient analysis. The transient data are interpreted using the expanded form of the Theis solution. The generalization for a test in which the pumping rate varies is derived using the principle of superposition:

$$s_w(t) = \frac{2.303}{4\pi T}\sum_{i=1}^{NP(t)} \Delta Q_i \, W\left(\frac{r_w^2 S}{4T(t-ts_i)}\right) + \frac{Q_t}{4\pi T} 2S_w + CQ_t^2 \qquad (28)$$

Here ts_i denotes the starting time of the i^{th} pumping step, ΔQ_i represents the change in the pumping rate at the start of this step, *NP(t)* represents the number of steps that have occurred up to the current time *t*, and Q_t is the current pumping rate at time *t*. These terms are illustrated in Figure 24.

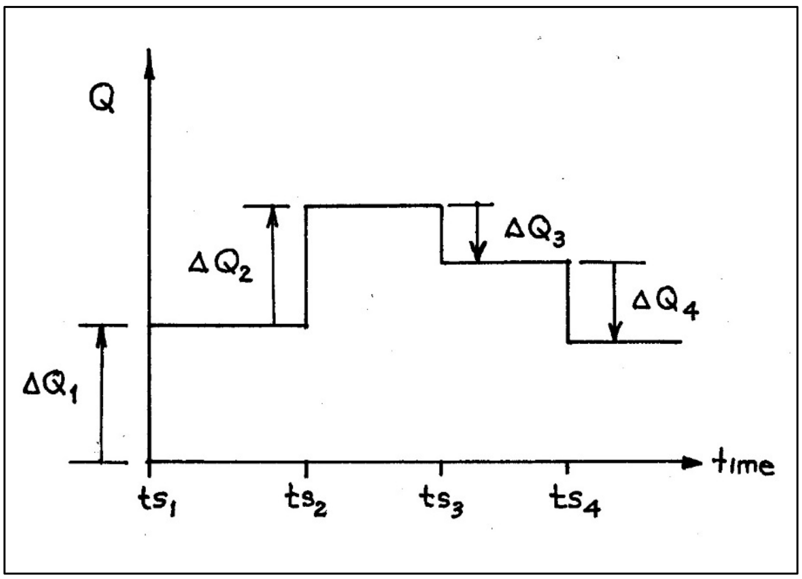


Figure 24. Schematic representation of time-varying pumping

The current pumping rate is related to the steps according to:

$$Q_t = \sum_{i=1}^{NP(t)} \Delta Q_i \qquad (29)$$

In practice, the data collected during a step test are matched with Equation (28) with a fitting routine, such as the nonlinear least-squares fitting routine incorporated in packages like AQTESOLV. The application of the transient analyses will be demonstrated with a case study but first we must include a note of caution regarding the analyses.

A caution on the interpretation of step tests: Parameter correlation

In Section 3 we considered an example of a pumping well that penetrated the full thickness of an ideal aquifer. The following parameters were specified for the example:

- Transmissivity, $T = 8.64$ m^2/d;
- Storativity, $S = 10^{-4}$;
- Dimensionless skin factor, $S_w = 0.5193$; and
- Nonlinear well loss coefficient, $C = 1.340\times10^{-4}$ m^{-5}d^2.

In the previous calculation, we assumed that the well was pumped at a constant rate. This time, let us assume that the well is pumped for three even steps according to the following schedule:

Elapsed time	Pumping rate, m^3/day
0 to 60 minutes	34.848
60-120 minutes	69.696
120-180 minutes	104.544

The drawdowns at the pumping well calculated with Equation (28) are plotted in Figure 25.

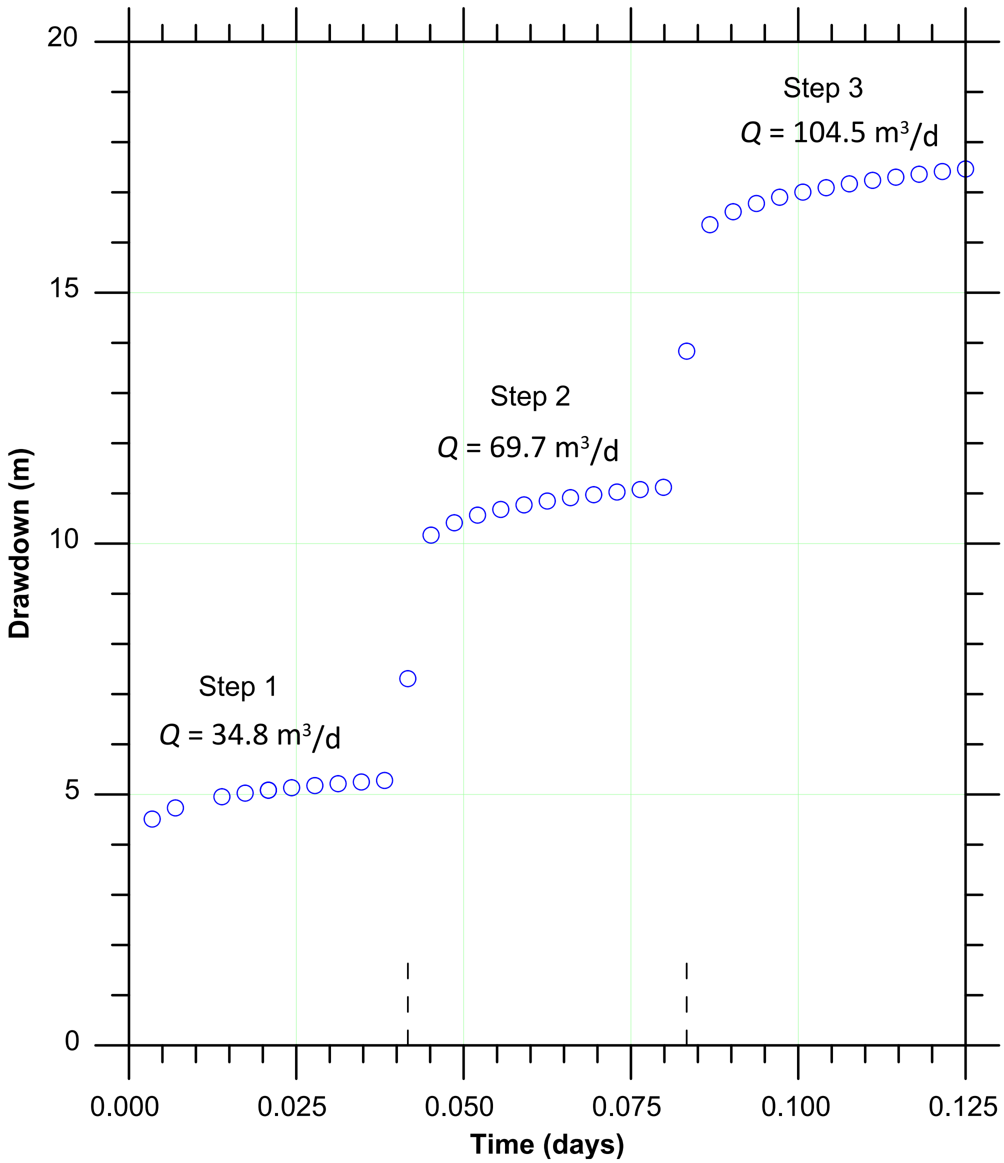


Figure 25. Pumping well drawdowns for a hypothetical step test

The analysis package AQTESOLV is used to fit the full transient record with Equation (25). To use a model that incorporates well losses, we must do two things with AQTESOLV:

- Tell AQTESOLV that we are interested in interpreting the drawdown as if it came from a pumping well and not an observation well. We do this by specifying the pumping well as an observation well at zero radial distance from the pumping well; and
- Choose the "Confined – Theis (1935) step drawdown test" solution.

How well does AQTESOLV do when we ask it to estimate simultaneously the transmissivity and storativity, T and S, *and* the well loss parameters C and S_w? The results of the automatic fit are shown in Figure 26.

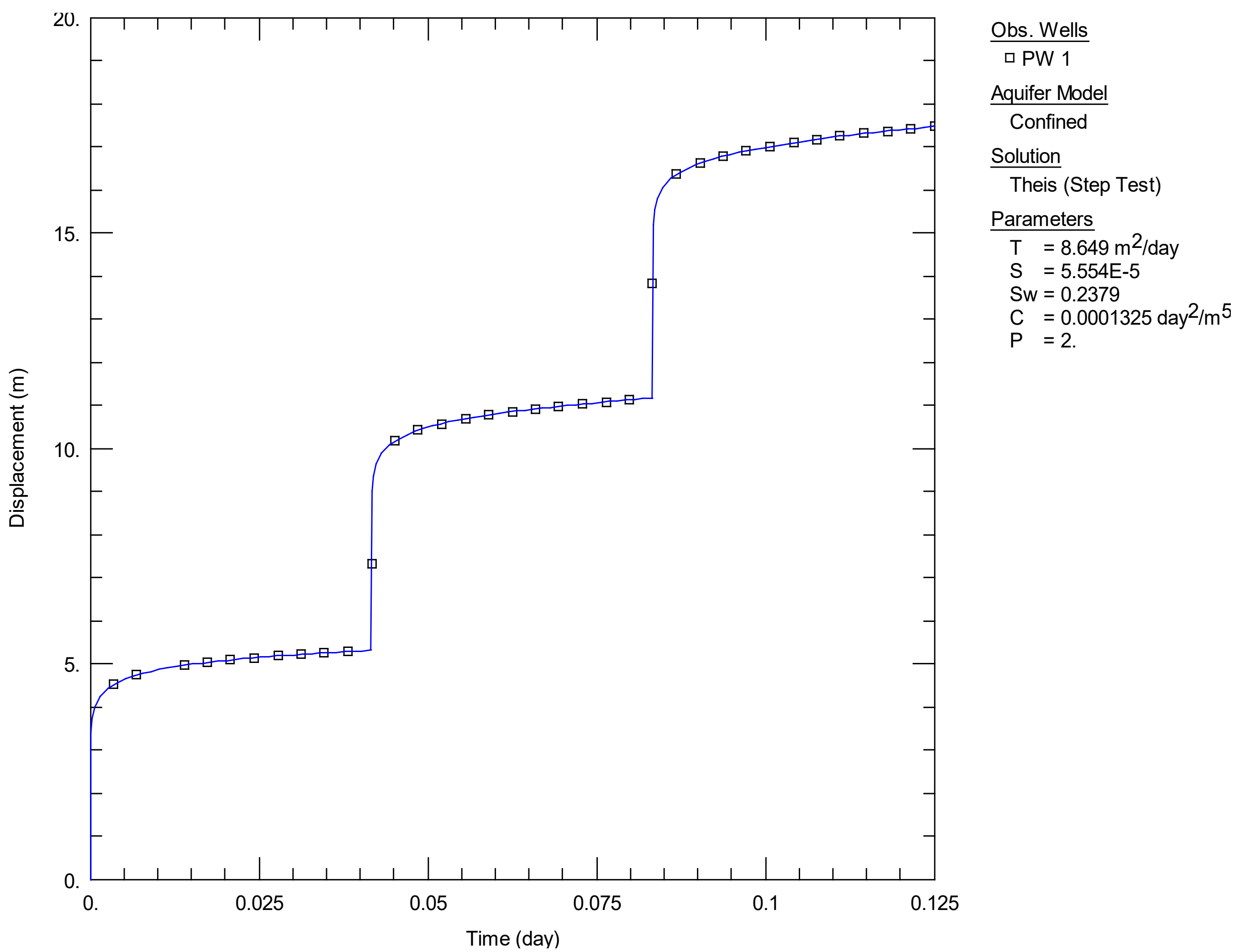


Figure 26. AQTESOLV match to hypothetical step test results

As shown in Figure 15, the drawdown data are matched very closely. The specified and fitted parameter values are listed below.

Parameter	Specified value	Fitted value
T	8.64 m^2/d	8.65 m^2/d
S	1.0×10^{-4}	5.55×10^{-5}
S_w	0.5193	0.2379
C	1.34×10^{-4} d^2/m^5	1.32×10^{-4} d^2/m^5
P	2.0 (fixed)	2.0 (fixed)

The fitted values of the transmissivity, T, and the nonlinear well loss coefficient, C, are very close to the values that were specified in the calculations. By pumping the well at more than one pumping rate, we increase our chances of obtaining unique estimates for these parameters. However, the storativity, S, and the skin loss coefficient, S_w, are significantly different from the specified values.

Why do we obtain an essentially perfect match to the drawdowns but with very different parameter values? Is it possible that the parameters estimated through the "objective" nonlinear least-squares fitting are not unique?

To assess whether the parameter values estimated for a particular analysis are unique, it is necessary to examine whether any of the fitted parameters are correlated. With the AQTESOLV software it is possible to examine parameter correlation, under the window labeled **diagnostics**. The report of the fitting for this example is reproduced in Figure 27. Reviewing the reported "Parameter Correlations", we see that the storage coefficient and the skin loss coefficient have Parameter Correlation values of 1.00. This means that the parameters are perfectly correlated.

DIAGNOSTICS REPORT

Diagnostic Statistics

Estimation complete! Parameter change criterion (ETOL) reached.

Aquifer Model: Confined
Solution Method: Theis (Step Test)

Estimated Parameters

Parameter	Estimate	Std. Error	Approx. C.I.	t-Ratio	
T	8.649	0.004118	+/- 0.008397	2100.2	m^2/day
S	5.554E-5	4.83E-5	+/- 9.848E-5	1.15	
Sw	0.2379	0.4298	+/- 0.8763	0.5536	
C	0.0001325	1.319E-7	+/- 2.689E-7	1004.7	day^2/m^5
P	2.	not estimated			

C.I. is approximate 95% confidence interval for parameter
t-ratio = estimate/std. error
No estimation window

K = T/b = 0.8649 m/day (0.001001 cm/sec)
Ss = S/b = 5.554E-6 1/m

Parameter Correlations

	T	S	Sw	C
T	1.00	-0.85	-0.85	0.31
S	-0.85	1.00	1.00	-0.21
Sw	-0.85	1.00	1.00	-0.21
C	0.31	-0.21	-0.21	1.00

Residual Statistics

for weighted residuals

Sum of Squares .... 4.392E-5 m^2
Variance .... 1.417E-6 m^2
Std. Deviation....... 0.00119 m
Mean .... -0.0001331 m
No. of Residuals 35
No. of Estimates 4

Figure 27. Diagnostic reports for the step test example

What does a Parameter Correlation value of 1.00 mean? In this case, it means that the storativity *S* and the dimensionless skin factor S_w are perfectly correlated. From the perspective of curve fitting, this means that the values of *S* and S_w *cannot* be estimated independently; that is, it is impossible to obtain unique estimates of *S* and S_w. As shown in Figure 16, if we fix *S* at two different values we obtain equally good matches to the observations with two different values of S_w.

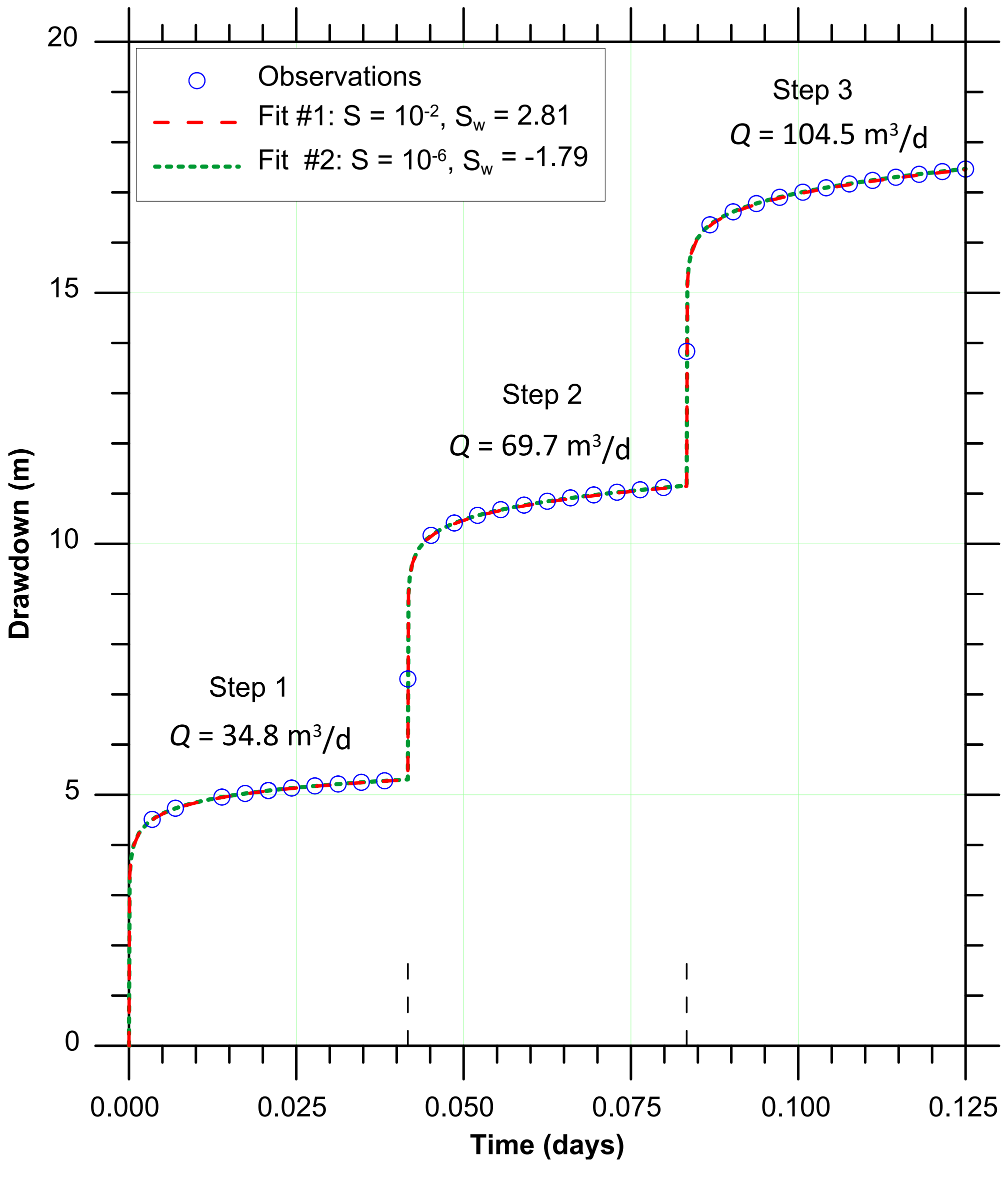


Figure 28. Alternative AQTESOLV matches to hypothetical step test results

To understand why the storativity and skin factor are perfectly correlated we return to the Cooper-Jacob solution with the addition of skin losses:

$$s(r_w, t) = s_{formation}(r_w, t) + \Delta s_{skin}$$

$$s_w(t) = \frac{Q}{4\pi T}\left[-0.5772 - \ln\left\{\frac{r_w^2 S}{4Tt}\right\}\right] + \frac{Q}{4\pi T}\, 2S_w$$

Making use of the properties of the log function, the solution can be re-arranged as:

$$s(r_w, t) = \frac{Q}{4\pi T}\left[-0.5772 - \ln\left\{\frac{r_w^2\, S}{4Tt}\right\} - \ln\{EXP\{2S_w\}\}\right]$$

Collecting terms:

$$s(r_w, t) = \frac{Q}{4\pi T}\left[-0.5772 - \ln\left\{\frac{r_w^2\, S\; EXP\;\{-2S_w\}}{4Tt}\right\}\right] \tag{30}$$

Defining an "effective" storage coefficient, S_E, as:

$$S_E = SEXP\{-2S_w\} \tag{31}$$

Equation (30) reduces to the Cooper-Jacob approximation with the actual storage coefficient S replaced by S_E:

$$s(r_w, t) = \frac{Q}{4\pi T}\left[-0.5772 - \ln\left\{\frac{r_w^2\, S_E}{4Tt}\right\}\right] \tag{32}$$

What do the paired values of S and S_w in Figure 28 have in common? For the original parameters and the parameter values shown in Figure 28, we calculate:

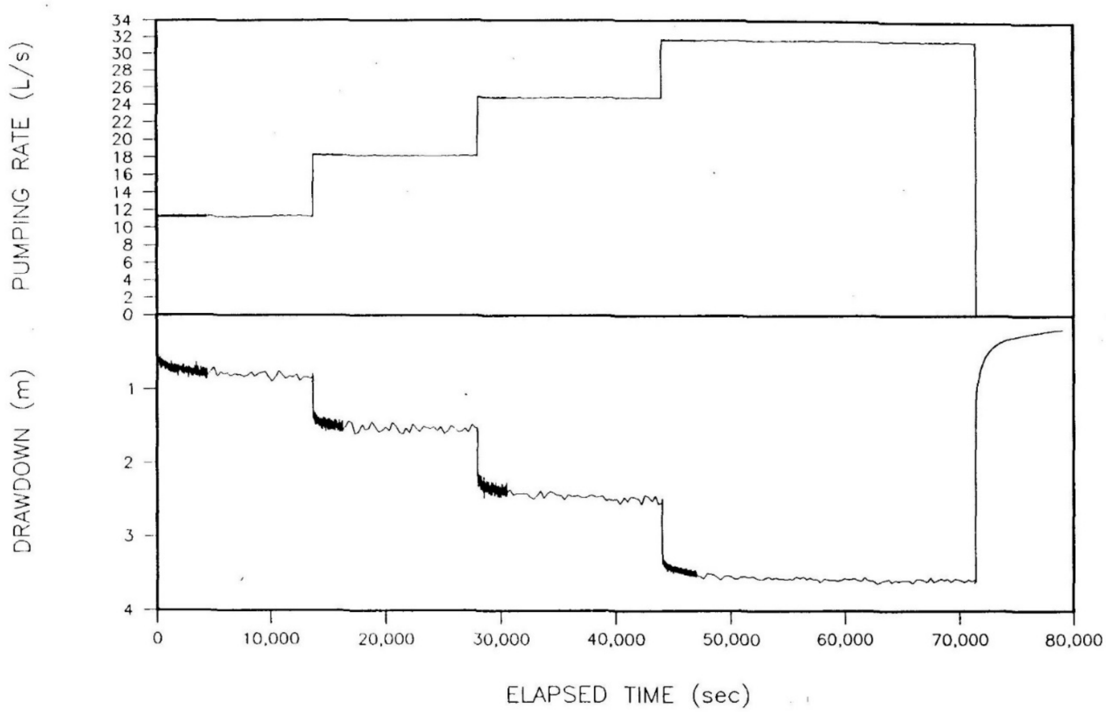
$$\begin{aligned} S \times EXP\{-2S_w\} &= (10^{-2}) \times EXP\{-2(2.81)\} = 3.6 \times 10^{-5} \\ &= (10^{-6}) \times EXP\{-2(-1.79)\} = 3.6 \times 10^{-5} \end{aligned}$$

The paired values of S and S_w yield identical values of $S{\times}EXP\{-2S_w\}$. These results demonstrate that it is possible to only know the product $S{\times}EXP\{-2S_w\}$.

Confined storage coefficients vary over a relatively narrow range, from about $1.0{\times}10^{-6}$ to $1.0{\times}10^{-4}$. Fitted values that are well outside of this range likely suggest the presence of a skin zone around the pumping well.

Case study: PW6/63, Guelph, Ontario – Part 2

The plot of the pumping history and drawdowns for the PW6/63 step test is reproduced below. The data are of sufficient quality to support a more rigorous transient analysis. With the results of the rigorous analyses in hand we can evaluate the reliability of the transmissivity estimates developed with the simpler approaches.



A computer-assisted analysis package is used to estimate the test data, matching the Theis solution to the drawdowns. The following decisions are made to constrain the analysis and avoid the non-uniqueness arising from parameter correlation:

- We assume a "physically realistic" value for the storage coefficient, *S*; of 1.0×10^{-5}; and
- We fix the value of the nonlinear well loss coefficient, *C*, based on the results from the Hantush-Bierschenk plot.

$$C = 0.00225\ \frac{m}{(\mathrm{L/s})^2}\left|\frac{1000\ L}{m^3}\right|^2 = 2{,}250\ \frac{m}{(m^2/\mathrm{s})}$$

The results of the computer-assisted analysis are shown in Figure 29. As shown in the figure, it is possible to match closely the entire time-drawdown record with a transmissivity of **3,760 m²/d**.

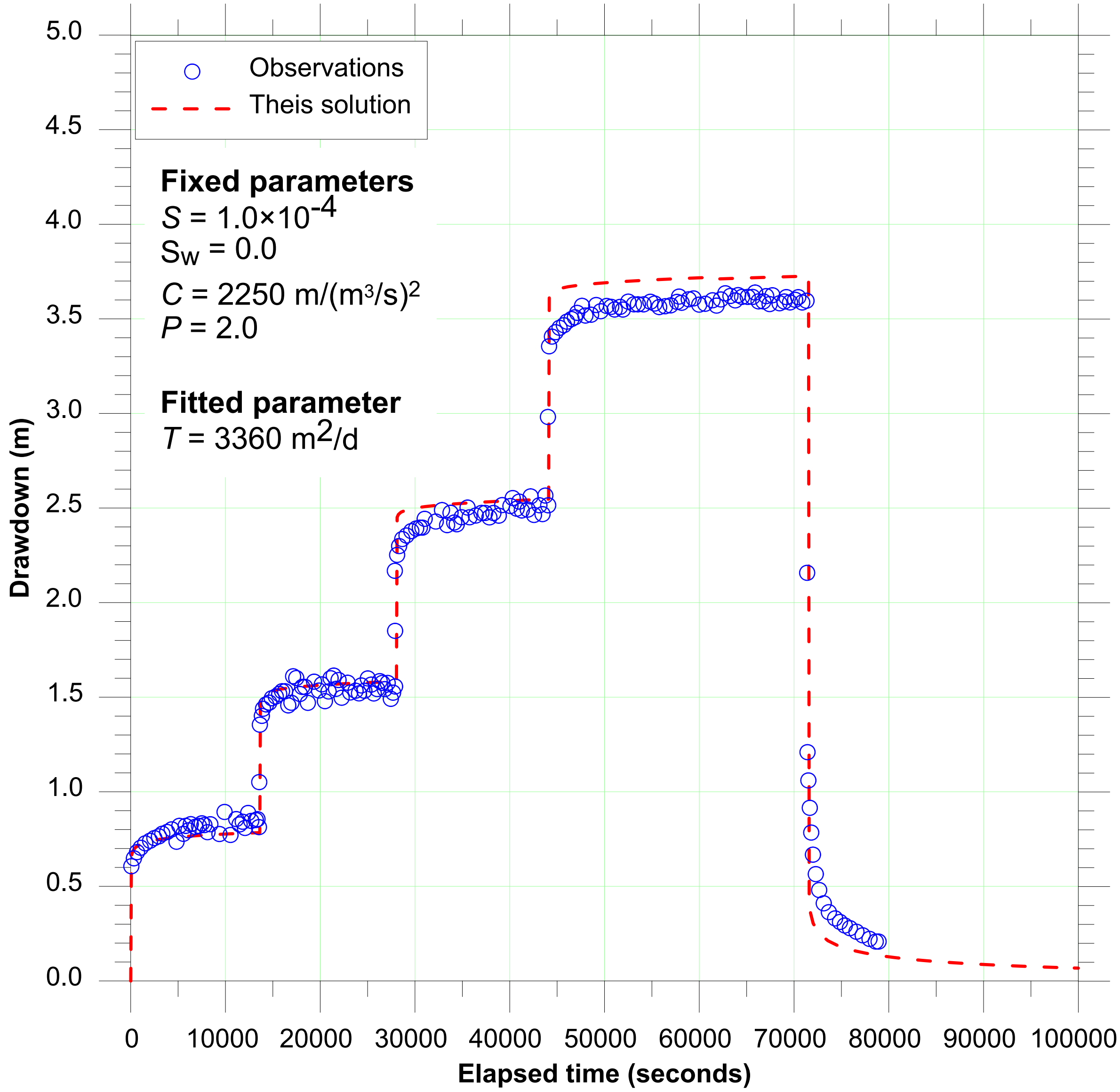


Figure 29. Match to the full transient drawdown record

Assessment

The advantage of estimating the transmissivity with the corrected specific capacity is that it is simple to do. Is the resulting estimate consistent with the estimate developed from a more rigorous analysis that considers all of the available data?

- Correlation with specific capacity: $\boldsymbol{T} \cong$ **3,020 m^2/d**; and
- Rigorous transient analysis: $\boldsymbol{T}$ **= 3,760 m^2/d**.

The transmissivity estimate obtained with the corrected specific capacity is relatively close to the estimate developed from the rigorous analysis. It is certainly consistent with our expectations regarding a "reconnaissance-level" estimate. The results of the simplified and rigorous analyses are in fact complementary. The preliminary estimate of the transmissivity derived with the estimate of the specific capacity with the nonlinear losses removed provides a useful check on our more rigorous analysis.

The results of the analyses highlight the importance of accounting for well losses in this case. The "raw" specific capacities range from 13.2 to 8.9 L/s/m. These are significantly smaller than the value estimated after the nonlinear well losses are removed, 25 L/s/m.

14. Synthesis of pumping well and observation well drawdowns

The consistency of the drawdowns for a pumping well and observation wells can best be assessed by plotting all of the drawdown data on a semilog *composite plot* (Cooper and Jacob, 1946). The composite plot has an axis of time/radius^2. According to the Theis conceptual model, the drawdowns at any point in the pumped aquifer will fall on a single line on a composite plot. However, we know that for most pumping wells there are head losses in addition to those that occur in the formation. If we assemble the data on a composite semilog plot the additional head losses in the pumping well will plot with a constant offset with respect to the observation wells. This concept is illustrated with some example calculations.

Let us consider a simple example involving a single pumping well that penetrates the full thickness of a homogeneous, horizontal, and perfectly confined aquifer of infinite extent. The aquifer is assumed to have a transmissivity and storativity of 8.64 m^2/day and 1.0×10^{-4}, respectively. The aquifer is pumped at a constant rate of 104.54 m^3/day. The observation well is located 10 m from the pumping well. The well is surrounded by a skin and there are nonlinear well losses. The additional well losses are characterized by the following parameters:

- $S_w = 0.5193$; and
- $P = 2$; and
- $C = 1.340\times10^{-4}$ m^{-5}d^2.

The results of the example calculations are assembled on a composite plot in Figure 30. The results do not approximate a single straight line. The drawdowns for the pumping well and observation appear to approximate parallel lines. The same slopes and offset of the pumping well and observation well drawdowns in Figure 30 are key diagnostic results. The parallel lines confirm that it is possible to estimate a representative bulk average transmissivity. The offset points to additional well losses in the pumping well drawdowns.

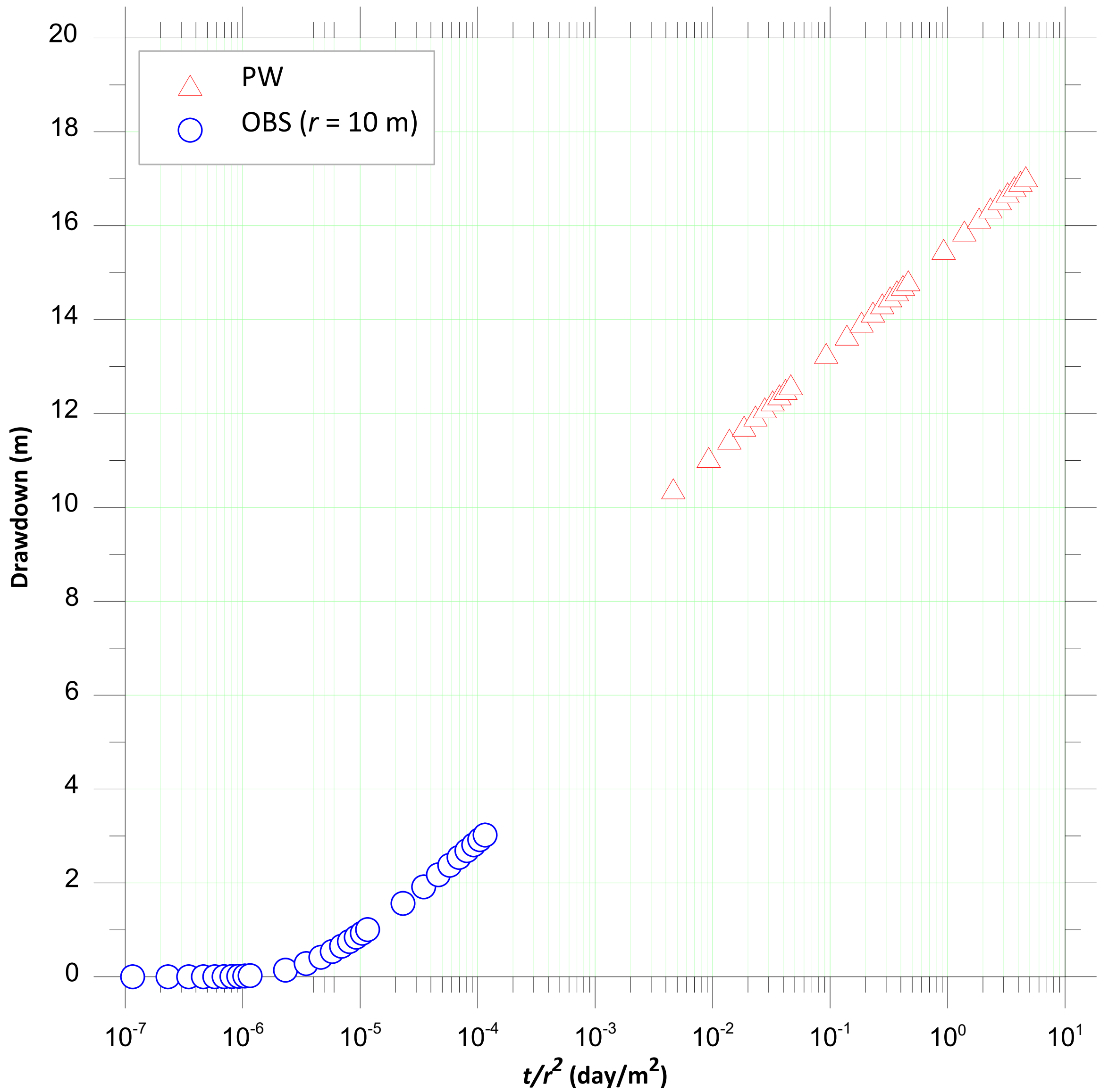


Figure 30. Composite plot for the pumping well and observation well

The Cooper-Jacob straight-line analyses are shown in Figure 31. The parallel slopes yield a consistent estimate of the transmissivity. Since the storage coefficient is estimated from the intercept of the straight lines fitted through the data, another important outcome of this plotting approach is that the Cooper-Jacob straight-line analyses of the individual records will yield different storage coefficients.

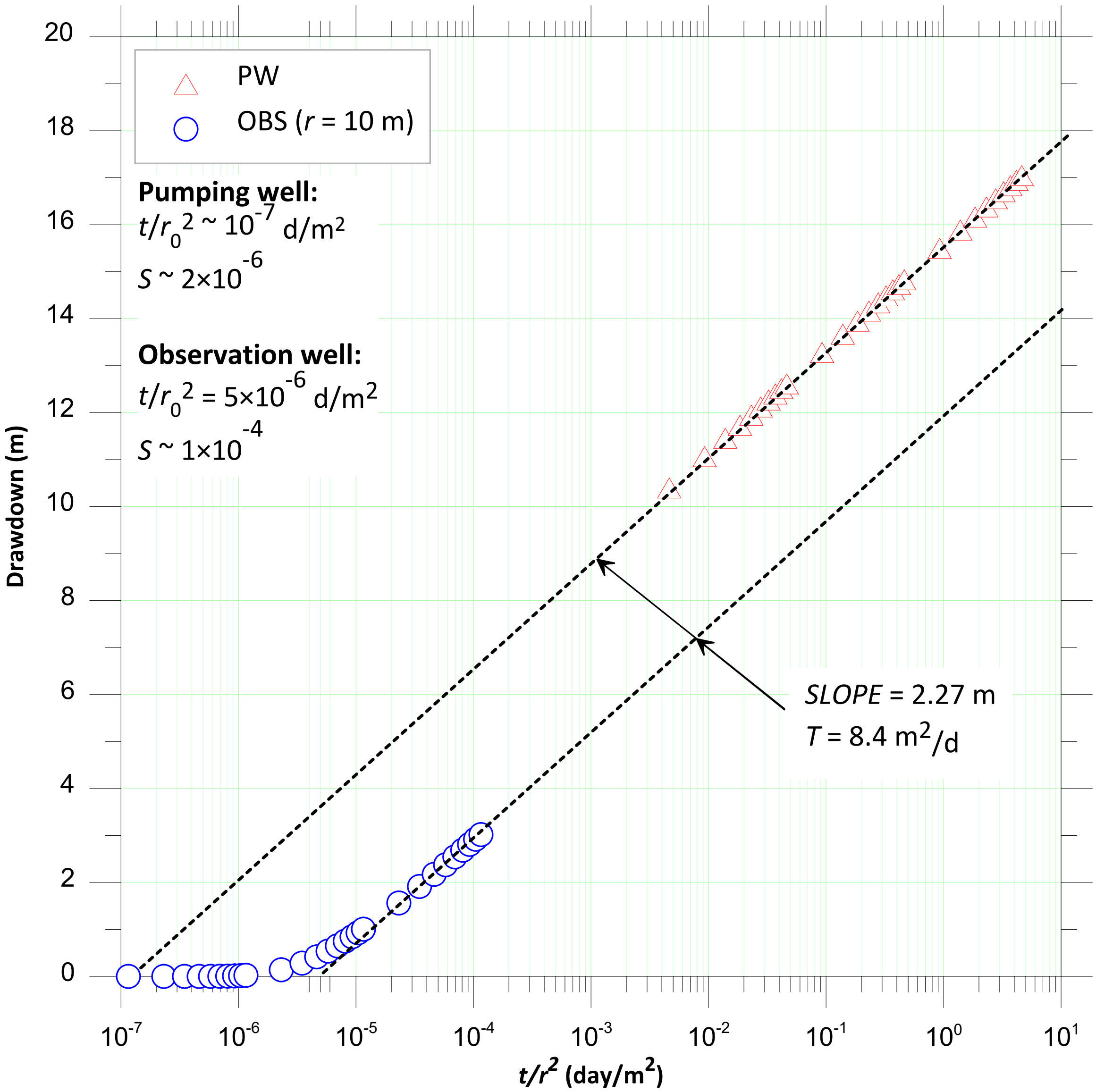


Figure 31. Cooper-Jacob analyses on the composite plot

As shown in Figure 31, the storage coefficient estimated for the observation well is 1.0×10^{-4} (as specified) and the storage coefficient for the pumping well is estimated to be 2.0×10^{-6}. The Cooper-Jacob solution can be used to explain why the data from the pumping well yields an inconsistent storage coefficient. If we replace the Theis well function with the Cooper-Jacob approximation, we obtain:

$$s_w(t) = \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{{r_w}^2 S}{4Tt}\right\}\right] + CQ^P + \frac{Q}{4\pi T}2S_w$$

Expanding the *ln* term and rearranging yields:

$$s_w(t) = \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r_w^2}{4Tt}\right\} - ln\{S\} + 2S_w + \frac{4\pi T}{Q}CQ^P\right]$$

Expanding using the properties of the log function yields:

$$s_w(t) = \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r_w^2}{4Tt}\right\} - ln\left\{S \times EXP\{-2S_w\} \times EXP\left\{-\frac{4\pi T}{Q}CQ^P\right\}\right\}\right]$$

We can write this in an equivalent form in terms of an "effective" storage coefficient as:

$$s_w(t) = \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r_w^2 S_{eff}}{4Tt}\right\}\right]$$

with:

$$S_{eff} = S \times EXP\{-2S_w\} \times EXP\left\{-\frac{4\pi T}{Q}CQ^P\right\}$$

Turbulence and skin effects confound estimation of reliable storativities from pumping well data. However, the inference of different storage coefficients for observation and pumping wells is actually useful. An inconsistent, and in some cases non-physical, estimate of the storage coefficient from the pumping well drawdowns is an important indicator that there are additional head losses in the pumping well.

15. Summary of key points

1. Head loss in the formation is only one of the processes that cause drawdowns in a pumping well.

2. If all we have is one point (Q,s_W) and we don't know much about the well, then estimating the transmissivity from the "raw" specific capacity may be the best we can do. This approach does not allow us to consider specific information we may have, such as the duration of pumping and well construction details, and the significance of well losses.
3. If a time history of pumping well drawdowns is available, and our only objective is to estimate transmissivity, we can begin and end with the Cooper-Jacob straight-line analysis.
4. If we want to know more about a pumping well than just the transmissivity in its vicinity, we require both a constant-rate pumping test and a step test. Step testing is the only definitive method of evaluating nonlinear well losses.
5. When interpreting pumping well drawdowns, we usually cannot estimate all parameter values. For example, we cannot estimate the storage coefficient and skin factor separately. Watch for non-physical parameter estimates and parameter correlation.

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Interpretation of pumping well drawdowns: Additional readings

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ESTIMATING THE TRANSMISSIBILITY OF AQUIFERS FROM THE SPECIFIC CAPACITY OF WELLS

By Charles V. Theis, Russell H. Brown, and Rex R. Meyer

ABSTRACT

The specific capacity of a well can be used as a basis for estimating the coefficient of transmissibility of the aquifer tapped by the well. From assumed values for the hydrologic constants of the aquifer, separate formulas including a term for specific capacity are developed for the transmissibility of water-table and artesian aquifers. From a chart relating the well diameter, the specific capacity of the well, and the coefficients of transmissibility and storage, the transmissibility of the aquifer can be estimated from the known specific capacity of the well or the specific capacity of the well can be estimated from the known transmissibility of the aquifer. These methods are subject to limitations but are useful means of approximation.

THE GENERAL RELATIONSHIP BETWEEN TRANSMISSIBILITY AND SPECIFIC CAPACITY

In many ground-water investigations, especially those of a reconnaissance type, the specific capacities of wells provide the only basis for estimating the transmissibility of the aquifers tapped by the wells. Generally speaking, high specific capacities indicate an aquifer having a high coefficient of transmissibility, T, and low specific capacities indicate an aquifer having a low T. However, a precise correlation between the specific capacities of wells and the T values of the aquifers they tap has not yet been established.

The specific capacity of a well cannot be an exact criterion of T in the vicinity of the well because, obviously, the yield of the well per foot of drawdown is also a function of other factors such as the diameter of the well, the depth to which the well extends into the aquifer, the type and amount of perforation in the well casing, and the extent to which the well has been developed. However, estimates of T that are based on the specific capacities of wells should be reasonably reliable and could be made without the elaborate tests necessary for precise determinations. Therefore, if developed within the limits of idealized assumptions, a formula expressing the theoretically exact relationship between the specific capacity of a well and the transmissibility of the aquifer which the well taps would be highly useful in the making of reconnaissance ground-water studies provided the theoretical formula is empirically modified for prevailing field conditions.

ESTIMATING THE TRANSMISSIBILITY OF A WATER-TABLE AQUIFER FROM THE SPECIFIC CAPACITY OF A WELL

By Charles V. Theis

The relation between the discharge of a well and the water-level drawdown a short distance from the well is given by an equation derived by Theis (1935). The value of u in that equation is small provided r is small, T and S are within the range of values for fairly productive aquifers, and t is at least several hours. For the purpose of this paper, the Theis formula can be written with negligible error as follows:

$$T=\frac{114.6Q}{s}\left[-0.577-\log_e\left(\frac{1.87r^2S}{Tt}\right)\right]. \tag{1}$$

The computation can be made somewhat simpler by substituting values for S and T that are within the range of fairly productive values for water-table aquifers. However, if corrections for these values are included, the formula remains general. Thus, if $T=1{,}000$ gpd per ft, $S=0.2$, and $t=1$ day, the formula for an average water-table aquifer corrected for variations of that aquifer from average is

$$T=\frac{114.6Q}{s}\left[-0.577-\log_e\left(\frac{1.87r^2\cdot 0.2}{100{,}000}\cdot\frac{S\cdot 100{,}000}{0.2\cdot T}\cdot\frac{1}{t}\right)\right]$$

$$=\frac{114.6Q}{s}\left[-0.557-\log_e\left(\frac{(3.74r^2\cdot 10^{-6})(5S)}{(T\cdot 10^{-5})t}\right)\right]$$

$$=-\frac{66Q}{s}+\frac{264Q}{s}\left[-\log_{10}(3.74r^2\cdot 10^{-6})-\log_{10}5S\right.$$

$$\left.+\log_{10}(T\cdot 10^{-5})+\log_{10}t\right].$$

Therefore,

$$T-\frac{264Q}{s}\log_{10}(T\cdot 10^{-5})=-\frac{66Q}{s}+\frac{264Q}{s}$$

$$[-\log_{10}(3.74r^2\cdot 10^{-6})-\log_{10}5S+\log_{10}t].$$

Let

$$T'=T-\frac{264Q}{s}\log_{10}(T\cdot 10^{-5}) \tag{2}$$

then

$$T'=-\frac{66Q}{s}+\frac{264Q}{s}[-\log_{10}(3.74r^2\cdot 10^{-6})-\log_{10}5S+\log_{10}t]$$

$$=\frac{Q}{s}[-66-264\log_{10}(3.74r^2\cdot 10^{-6})-264\log_{10}5S+264\log_{10}t].$$

Let

$$K=-66-264\log_{10}(3.74r^2\cdot 10^{-6}) \tag{3}$$

then

$$T'=\frac{Q}{s}(K-264\log_{10}5S+264\log_{10}t). \tag{4}$$

Values of K, computed for selected values of r, are as follows:

r (ft)	K	r (ft)	K
0.25	1,684	20	680
.50	1,524	30	588
1.0	1,367	40	521
5.0	996	50	469
10	838		

The foregoing formulas indicate the importance of both the storage coefficient and the duration of pumping when the coefficient of transmissibility is estimated from a single measurement of drawdown in an observation well. If $S=0.2$, the influence of the S term is zero because the formula was derived on that basis. However, if $S=0.1$, the S term would equal $-264\log_{10}5S=-264\log_{10}0.5=80$, or about 8 percent of the constant, K, for $r=5$ feet, and if $S=0.3$, the S term would equal -45, or about -4.5 percent of the same value for K. Provided S is known, the correction can be made, but if S is unknown, the error for a water-table aquifer (for which S ranges from 0.1 to 0.3) probably will be smaller than the errors inherent in the method. Although the correction for the duration of pumping also is comparatively small, it presumably should be made if, as in many cases, the duration is known. For an artesian aquifer, S is very small and the S term correction will be large, making it inadvisable to apply the formula (in its present form) for artesian conditions; for if $S=0.001$, the S term would be about double K for $r=5$ feet.

The coefficient of transmissibility cannot be determined explicitly from the computed values of T'. However, from charts giving the values of T' for various values of T and Q/s, the value of T can be ascertained from known values of T' and Q/s. Such a chart is shown in figure 99.

Thus, within the limits of the idealized assumptions, the coefficient of transmissibility of a water-table aquifer apparently can be computed without great error from a single measurement of drawdown in an observation well that is a short distance from a pumped well, even if the coefficient of storage is not known. However, the information generally available concerns the specific capacity of the pumped well. In the foregoing formulas Q represents the discharge of the well and s is the drawdown in a nearby observation well at a distance r from the pumped well. Obviously, the drawdown in the pumped well bears a relationship to the drawdown a short distance from the well. If this relationship can be ascertained approximately,

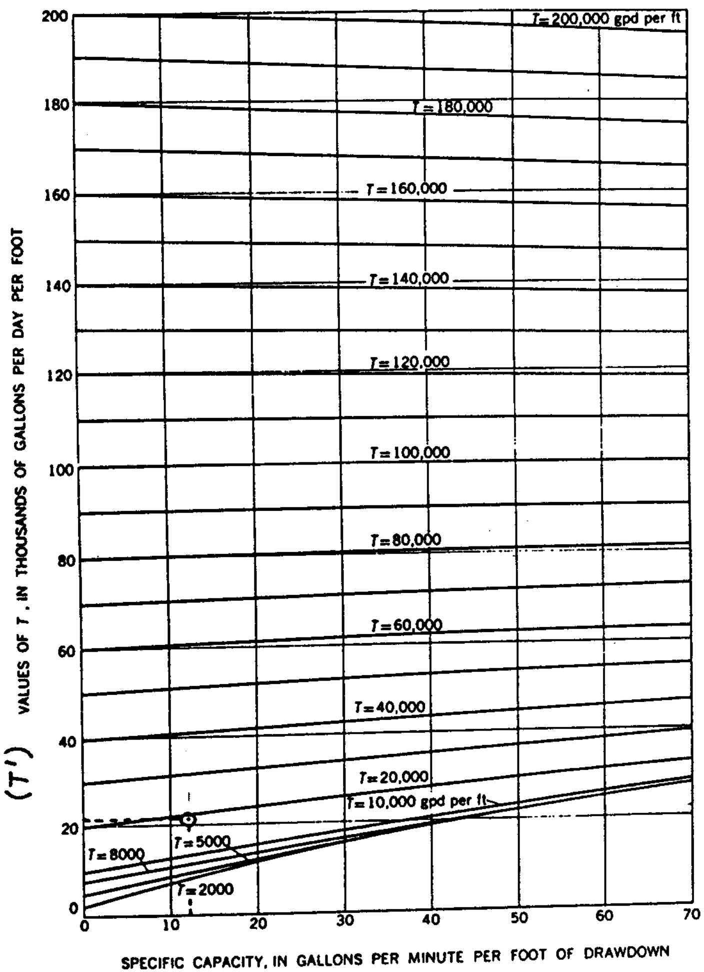

FIGURE 99.—Diagram for estimating the transmissibility of an aquifer from the specific capacity of a well.

the specific capacity of the pumped well can be substituted for the quantity Q/s for the appropriate distance from the well.

For small-diameter uncased wells that tap consolidated water-bearing rocks, or at least for wells that produced no sand or silt when developed, the distance r probably can be equated to the radius of the well. For instance, for a well 6 inches in diameter,

$$T' = C(1{,}684 - 264 \log_{10} 5S + 264 \log_{10} t),$$

in which

$$C = \frac{Q}{s} = \text{the specific capacity of the pumped well.}$$

In wells having perforated casing and for which no improvement in performance was noted upon development, some head is lost as the water moves through the perforations in the casing. The amount of head lost in this manner ranges widely according to whether or not the casing fits snugly against the wall of the hole. If it does, the drawdown within the aquifer at the wall of the hole presumably would be considerably less than within the well itself, and the specific capacity computed on the basis of the lesser drawdown would be considerably higher. An arbitrary increase, then, in the specific capacity probably would be justified for the computation of the coefficient of transmissibility. In consolidated formations in which the wall of a hole is rough and the casing does not fit tightly, the loss in head presumably is small and can be disregarded.

Many wells of large yield tap aquifers that consist of consolidated sand or gravel. Such wells yield readily to development and once they are developed the pumping level of the water both within and immediately outside the casing is generally higher than it would have been had they not been developed. It is difficult to estimate the extent to which the transmissibility of the materials in the immediate vicinity of a well has been increased by the development of the well. However, available data indicate that in many cases the effect is the same as if the well were 10 feet in diameter but had not been developed. Therefore, 996 (the factor for $r=5$ ft) would be a reasonable value to substitute for K in the equation for T'.

Although many empirical data should be gathered as to the relation between the specific capacities of wells and the transmissibilities of the tapped aquifers before any final correlation is made, present knowledge seems to justify the following equation for wells that have a diameter of about 1 foot and that tap water-table aquifers consisting of unconsolidated sediments:

$$T' = C(1 \pm 0.3)(1{,}300 - 264 \log_{10} 5S + 264 \log_{10} t).$$

The factor (1 ± 0.3) should be adjusted upward for wells having a small diameter, for wells that are poorly developed, and for wells with poorly perforated casing, and downward for larger and well-developed wells.

ESTIMATING THE TRANSMISSIBILITY OF AN ARTESIAN AQUIFER FROM THE SPECIFIC CAPACITY OF A WELL

By RUSSELL H. BROWN

The use of figure 99 can be demonstrated by the following example. Assume that examination of well logs and related data has led to an estimate of 0.15 as a likely coefficient of storage, S, for a given water-table aquifer, that a review of well records has revealed a number of completion (or acceptance) tests, and that data taken from the best controlled test show, for a 30-hour pumping period, the specific capacity of a 6-inch well to be 12 gpm per ft of drawdown. The order of magnitude of the coefficient of transmissibility is to be determined. From the preceding discussion by Theis,

$$
\begin{aligned}
T' &= \frac{Q}{s}(K-264 \log_{10}5S+264 \log_{10}t) \\
&= 12(1{,}684-264 \log_{10}0.75+264 \log_{10}1.25) \\
&= 12(1{,}684+33+26) \\
&= 20{,}900.
\end{aligned}
$$

As shown by figure 99, the abscissa of $T'=20{,}900$ gdp per ft intersects the ordinate of specific capacity equals 12 gpd per ft of drawdown about where $T=19{,}000$ gpd per ft. If S should later prove to be 0.25 instead of 0.15, the revised value of T' would be 20,200 and, from the chart, T would be about 18,000 gpd per ft. Thus it is evident that even large differences in S do not materially affect the value of T and that exercising judgment in selecting a value for S will produce results of the correct order of magnitude.

As stated by Theis (p. 333), the formulas and related constants derived by him are not applicable to artesian conditions. The principal objection in attempting to extend their application from water-table conditions to artesian conditions is the large adjustment in the K factor that becomes necessary if, for example, $S=2\times10^{-4}$, which is one-thousandth the assumed $S=0.2$. However, a formula and set of constants for artesian conditions can be found by paralleling the Theis derivation and using an assumed coefficient of storage of 2×10^{-4}. If it is assumed again that $T=100{,}000$ gpd per ft, Theis' diagram (fig. 99) can be used without modification.

If $T=100{,}000$ gpd per ft and $S=2\times10^{-4}$, then from equation 1 on page 332

$$
\begin{aligned}
T &= \frac{114.6Q}{s}\left[-0.577-\log_e\left(\frac{1.87r^2\cdot 2\cdot 10^{-4}}{100{,}000}\cdot\frac{S\cdot 100{,}000}{2\cdot 10^{-4}T}\cdot\frac{1}{t}\right)\right] \\
&= \frac{114.6Q}{s}\left[-0.577-\log_e\left(\frac{(3.74r^2\cdot 10^{-9})(5S\cdot 10^{3})}{(T\cdot 10^{-5})t}\right)\right] \\
&= \frac{66Q}{s}+\frac{264Q}{s}[-\log_{10}(3.74r^2\cdot 10^{-9}) \\
&\qquad -\log_{10}(5S\cdot 10^{3})+\log_{10}(T\cdot 10^{-5})+\log_{10}t].
\end{aligned}
$$

Therefore,

$$
T-\frac{264Q}{s}\log_{10}(T\cdot 10^{-5})=-\frac{66Q}{s}+\frac{264Q}{s}
$$

$$
[-\log_{10}(3.74r^2\cdot 10^{-9})-\log_{10}(5S\cdot 10^{3})+\log_{10}t].
$$

Again let

$$
T'=T-\frac{264Q}{s}\log_{10}(T\cdot 10^{-5}).
$$

Then

$$
\begin{aligned}
T' &= -\frac{66Q}{s}+\frac{264Q}{s}[-\log_{10}(3.74r^2\cdot 10^{-9})-\log_{10}(5S\cdot 10^{3})+\log_{10}t] \\
&= \frac{Q}{s}[-66-264\log_{10}(3.74r^2\cdot 10^{-9})-264\log_{10}(5S\cdot 10^{3})+264\log_{10}t].
\end{aligned}
$$

Let

$$
K=-66-264\log_{10}(3.73r^2\cdot 10^{-9}). \tag{5}
$$

Then

$$
T'=\frac{Q}{s}[K-264\log_{10}(5S\cdot 10^{3})+264\log_{10}t]. \tag{6}
$$

Values of K, computed for selected values of r, are as follows:

r (ft)	K	r (ft)	K
0. 25	2, 477	20	1, 472
. 50	2, 318	30	1, 379
1. 0	2, 159	40	1, 313
5. 0	1, 790	50	1, 262
10	1, 633		

If the value of S is as large as 2×10^{-3} (10 times the assumed value) the effect will be to decrease K for $r=5$ feet by nearly 15 percent. For larger values of K this percentage obviously is lower, and for smaller values it is higher. Conversely, if S is as low as 2×10^{-5} (one tenth the assumed value) the effect will be to increase K by nearly 15 percent.

The application of the equation derived for T' for artesian conditions can be demonstrated by an example. Assume that the best estimate of S for a given artesian aquifer is 4×10^{-5}. Furthermore, data collected during a 30-hour acceptance test of a 6-inch well show that the specific capacity of the well is 7.5 gpm per ft of drawdown. The coefficient of transmissibility may be computed by following the same procedure used in the previous example.

Thus,

$$
\begin{aligned}
T' &= \frac{Q}{s}[K-264\log_{10}(5S\cdot10^{3})+264\log_{10}t] \\
&= 7.5(2{,}477-264\log_{10}0.2+264\log_{10}1.25) \\
&= 7.5(2{,}477+184+26) \\
&= 20{,}200.
\end{aligned}
$$

According to figure 99, $T=18{,}000$ gpd per ft (approx.) where the ordinate of 7.5 intersects the abcissa of 20,200. If it later develops that a value of 4×10^{-4} is a better estimate of S, then T' would be 18,200 and T would be about 16,000 gpd per ft.

A CHART RELATING WELL DIAMETER, SPECIFIC CAPACITY, AND THE COEFFICIENTS OF TRANSMISSIBILITY AND STORAGE

By Rex R. Meyer

The relationships of well diameter, specific capacity, and the coefficients of transmissibility, T, and storage, S, are shown graphically in figure 100. This graph was prepared by (1) computing, for various values of T and S, the theoretical drawdown in wells having diameters of 6, 12, and 24 inches, (2) computing the specific capacity of those wells (on the assumption that they are 100 percent efficient), and (3) plotting the specific capacity against S to form a family of curves which represent the different values of T. For the sake of clarity, the curves for a well 24 inches in diameter were not plotted in the upper part of the graph; they would be virtually parallel to the curves for a well 12 inches in diameter and lie above them at a distance equal to that between the curves for wells 6 inches and 12 inches in diameter. The specific capacity at the end of 1 day's pumping is shown on the left scale of the graph. The values of S, shown on the bottom scale, range from those for artesian conditions on the left to those for water-table conditions on the right. Each group of curves for a specific T is bracketed on the right margin.

Figure 100 can be used to determine the approximate T of an aquifer if the specific capacities of wells are the only available data. It also can be used to determine the approximate specific capacity of a well which is to be drilled into an aquifer for which T and S are known. The computed theoretical specific capacity is useful not only for planning purposes but also, when compared to the specific capacity determined from a field test, as a means of determining the approximate efficiency of a well. Although determinations made from figure 100 may not be exact owing to unknown factors that must be estimated, the graph serves as a measure for approximation.

A cursory study of the graph reveals that it has certain limitations. One of the principal factors affecting the specific capacity of a well is the entrance loss of the water. The graph is based on the assumption that the wells are 100 percent efficient or, in other words, that when the wells are pumped the water level inside and immediately outside the casing or screen is the same. Because, in most wells, the water level immediately outside is higher than inside, the observed specific capacity is somewhat less than that of an ideal well. The specific capacity of a well is affected also by the diameter of the well. The well diameters shown on the graph—6, 12, and 24 inches—are considered to be the effective diameters of the wells. If an aquifer is composed of consolidated rocks, the effective diameter probably is approximately the same as the diameter of the well. However, if the material in an aquifer consists of unconsolidated materials and if

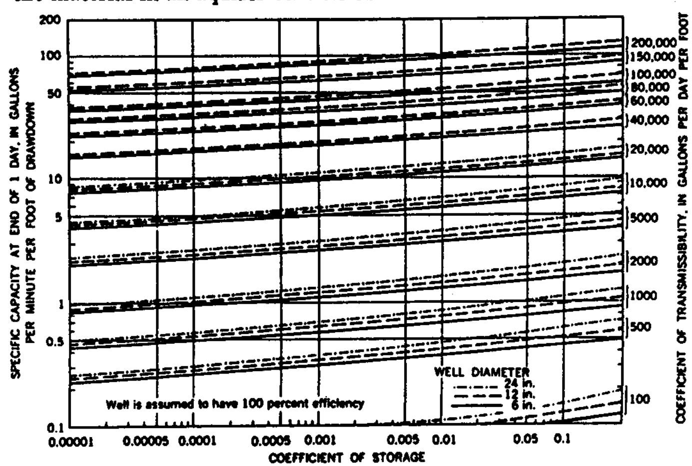

FIGURE 100.—Graph showing relation of well diameter, specific capacity, and coefficients of transmissibility and storage.

the well has been highly developed, the effective diameter may be substantially larger than the diameter of the screen. On the other hand, a seemingly highly developed well may be very inefficient because of caving or faulty construction, and, accordingly, have an effective diameter less than the diameter of the screen. Other conditions being the same, a change in the effective diameter has the greatest effect on the specific capacity of wells in aquifers that have a low T and a high S.

The graph shows that large changes in S correspond to relatively small changes in T and specific capacity; therefore, inaccuracy in estimating S generally is not a serious limiting factor. Moreover, from a general knowledge of the geology and hydrology, an aquifer usually can be classified as principally water table or artesian, and S can be estimated accordingly. However, the graph should not be used in an attempt to determine S even when accurate values of the specific capacity and T are available.

If the pumped well taps less than the full thickness of the aquifer—thus introducing vertical components of flow—or if it taps a thin water-table aquifer so that the water-level drawdown is a substantial fraction of the original saturated thickness, the graph obviously cannot be applied without serious error.

The time interval of 1 day used for computing the specific capacity scale on the graph was selected arbitrarily. An error will be introduced if the specific capacity determined in the field is based on a shorter or longer period of pumping. The amount of the error is small for high values of T and low values of S but increases substantially for low values of T and high values of S.

The procedure for using the log graph to determine T from the specific capacity of a well is as follows:

1. Select the specific capacity on the left margin.
2. Move horizontally along the abcissa to the intersection of the ordinate through the estimated value of S.
3. From this intersection move along a curve or parallel to the family of curves, and find the value of T on the right margin.

Although the specific capacity at the end of 1 day's pumping can be computed for an ideal well tapping an aquifer having known values of T and S, it can be determined more easily and quickly from the graph. To determine the theoretical specific capacity of such a well, the procedure described above is reversed; move left along or parallel to the curve from the known value of T to the intersection of the ordinate through the known value of S; thence move horizontally to the left margin and read the specific capacity.

If the graph is used with an understanding of its limitations, it should provide a useful tool in ground-water studies.

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GROUND WATER

COMPUTER NOTES

A COMPUTERIZED TECHNIQUE FOR ESTIMATING THE HYDRAULIC CONDUCTIVITY OF AQUIFERS FROM SPECIFIC CAPACITY DATA

by Kenneth R. Bradbury[a] and Edward R. Rothschild[b]

Abstract. Specific capacity data obtained from well construction reports can provide useful estimates of hydraulic conductivity (K). A simple computer program has been developed which can correct specific capacity data for partial penetration and well loss and, using an iterative technique, provide rapid estimates of K at hundreds of data points. The program allows easy data handling and is easily linked with existing statistical programs or contour mapping routines. The method was tested at two field sites in Wisconsin, one underlain by a sandy outwash aquifer, the other by fractured dolomite. In both areas, estimates of K from corrected specific capacity data agree reasonably well with data from pumping tests.

Introduction

Hydrogeologists continually seek and test simple, quick, and inexpensive methods for determining aquifer characteristics. The use of specific capacity tests to determine transmissivity (T), and ultimately hydraulic conductivity (K), is one such tool. Although the use of specific capacity data in estimating aquifer parameters is certainly not new (Theis *et al.*, 1963; Lohman, 1972), commonly used estimation techniques (described below) are somewhat slow and cumbersome. In this paper we describe a computer program which rapidly and accurately provides estimates of aquifer transmissivity at hundreds of points where specific capacity data are available, and we demonstrate that the technique gives excellent results at two field sites in Wisconsin. Because the solution is performed with the use of a computer, data can be manipulated easily and linked with available graphical and statistical packages.

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A specific capacity test involves pumping a well (of known construction) at a known rate and period of time, and measuring the drawdown within the well at the end of the test period. The length of the test is determined by how long it takes for the water level in the well to reach a state of apparent equilibrium, that is, when the change in drawdown is minimal with time. Specific capacity is defined as the discharge divided by the drawdown in the well, and the units generally used are gallons per minute per foot of drawdown (GPM/FT).

Theis *et al.* (1963) present a method of estimating transmissivity from specific capacity. They treat a specific capacity test as a short nonequilibrium pumping test, and utilize a graphical solution to estimate transmissivity. Several other workers, including Walton (1970), Lohman (1972), and Gabrysch (1968) have applied Theis' method to field problems. In this study, we replace the graphical approach with a short computer program utilizing an iterative procedure.

Estimating T from specific capacity involves a series of assumptions. These assumptions include a known storage coefficient (S), minimal well loss, full penetration, and a nonleaky, homogeneous and isotropic, artesian aquifer of infinite areal extent. (These assumptions are essential to use of the Theis equation, and are described in many basic texts.) Fortunately, because specific capacity varies with the logarithm of l/S, the solution is not very sensitive to variations in S, which can be estimated with sufficient accuracy from previous studies in an area, or by using representative values for a given aquifer type. If appropriate data are available, well loss corrections can be made. Corrections for partial penetration may be very important because few wells fully penetrate an aquifer. A method adopted from Brons and Marting (1961) is used in this study to correct for partial penetration.

To demonstrate the method, specific capacity data were used to estimate hydraulic conductivities for aquifers in two large field areas in Wisconsin (see Figure 1). One aquifer is a confined, fractured dolomite (area A), and the other consists of unconfined, unconsolidated sands and gravels (area B). In Wisconsin, specific capacity tests are generally performed by drillers at the time of well installation. Reports of the tests, as well as geologic logs and well construction reports for most wells are available at the Wisconsin Geological and Natural History Survey. In this study, we use available information to determine aquifer transmissivity, corrected for partial penetration of the wells, and

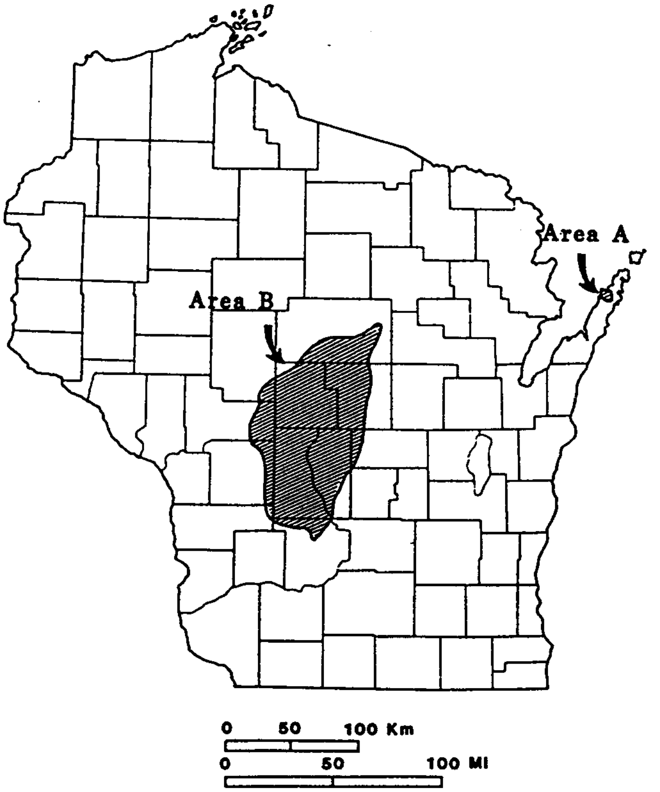

Fig. 1. Map of Wisconsin showing locations of field areas A (fractured dolomite) and B (sand and gravel).

then produce maps of hydraulic conductivity. The maps agree well with the more limited data available from pumping tests.

There are many advantages of using specific capacity information to compute hydraulic conductivity. The data are generally readily available and abundant: for area A, 224 specific capacity tests were available versus 5 pumping tests; for area B, 268 specific capacity tests were available versus 11 pumping tests. Estimates of hydraulic conductivity, based on specific capacity data, are quick, easy, and inexpensive, and when used in conjunction with limited pumping test data, may be the best method for mapping aquifer characteristics over large areas.

Computer Program Development

Theis *et al.* (1963) describe a method for estimating the transmissivity of an aquifer from the specific capacity of wells. Their analysis is based on the Jacob equation, given in consistent units as:

$$T = \frac{Q}{4\pi s} \ln \left(\frac{2.25\ Tt}{r_w^2 S} \right) \qquad (1)$$

where

T = transmissivity (L^2/t),

Q = discharge (L^3/t),

s = drawdown in the well (L),

t = pumping time (t),

S = storage coefficient (dimensionless), and

r_w = radius of the well (L).

Because T appears twice, this formula cannot be solved directly, and Theis *et al.* (1963) and Walton (1970) (among others) propose graphical solutions involving matching the specific capacity data to a family of curves. The graphical methods have the disadvantage of requiring a different set of curves for every possible combination of well radius, pumping period, and storage coefficient. In addition, any corrections for partial penetration or well loss require additional calculations.

Well loss is an increase in drawdown in the well bore over drawdown in the aquifer adjacent to the well. It is due to turbulent flow as water enters the well bore and pump, and depends on the pumping rate, construction of the well, and hydraulic properties of the tested aquifer. It is possible to correct specific capacity data for well loss using the equation (Csallany and Walton, 1963):

$$S_w = CQ^2 \qquad (2)$$

where

S_w = well loss (L),

C = well loss constant (t^2/L^5), and

Q = discharge (L^3/t).

Csallany and Walton present an equation with which to evaluate C from step-drawdown data.

Most private wells penetrate less than the full thickness of aquifers. During a specific capacity test, partially penetrating wells may yield anomalously low values of specific capacity, depending on the ratio of penetration (L) to aquifer thickness (b). In Wisconsin, the L/b ratio is sometimes as low as 0.1. Thus, a correction for partial penetration is necessary before estimating transmissivity from specific capacity. For unsteady drawdown in a partially penetrating well, Sternberg (1973) shows that

$$s = \frac{Q}{4\pi T} \left[\ln \left(\frac{2.25\ Tt}{r_w^2 S} \right) + 2\ s_p \right] \qquad (3)$$

where s_p is a "partial penetration factor" given by Brons and Marting (1961) as

$$s_p = \frac{1 - L/b}{L/b} \left(\ln \frac{b}{r_w} - G\{L/B\} \right) \qquad (4)$$

where

b = aquifer thickness (L),

L = length of open interval (L), and

G = a function of the L/b ratio.

Brons and Marting evaluate G(L/b) for various values of (b/r_w). In the present study the polynomial equation

$$G\{L/b\} = 2.948 - (7.363\ L/b) + 11.447\ \{L/b\}^2 - 4.675\ \{L/b\}^3 \qquad (5)$$

was fitted to the data of Brons and Marting by multiple regression, with a correlation coefficient of 0.992. Rewriting equation (3) to incorporate equation (2), we have

$$T = \frac{Q}{4\pi(s - s_w)}\left[\ln\left(\frac{2.25\ Tt}{r_w^2 S}\right) + 2\ s_p\right] \qquad (6)$$

The solution of equation (6) yields an estimate of T which is corrected for well loss and partial penetration, and incorporates t, S, and r_w.

Figure 2 shows a flow chart for a computer program which solves equation (6). The program first reads the data in the inconsistent units (gallons per minute, inches, feet, etc.) which are customarily used on driller's logs. After converting to consistent units (feet, seconds), the program solves equations (2), (4), and (5) directly. It then solves equation (6) iteratively, using an initial estimate of T (TGUESS) to calculate an updated estimate (TCALC). The program then substitutes the updated estimate for the original guess, and repeats the process until TGUESS and TCALC agree within a small error criterion (ERR). Finally the program prints the results.

Appendix A is a simple BASIC computer code written for an Apple IIe computer illustrating the estimation technique for a single well. A sample output is included in Appendix B. In practice, we expand this program to do several hundred estimations. The program is easily modified to change the types and methods of input and output. Currently it is designed to accept input either interactively or via a data file that has been merged with the program file. By including well coordinates in the input data, the output can be used directly in graphics plotting packages, as well as in statistical routines. The variables ERR and TGUESS have been assigned values of 0.1E-5 and 0.1, respectively. These can be altered by changing lines 300 and 320 of the program. The program also has been written in FORTRAN and is available from the authors.

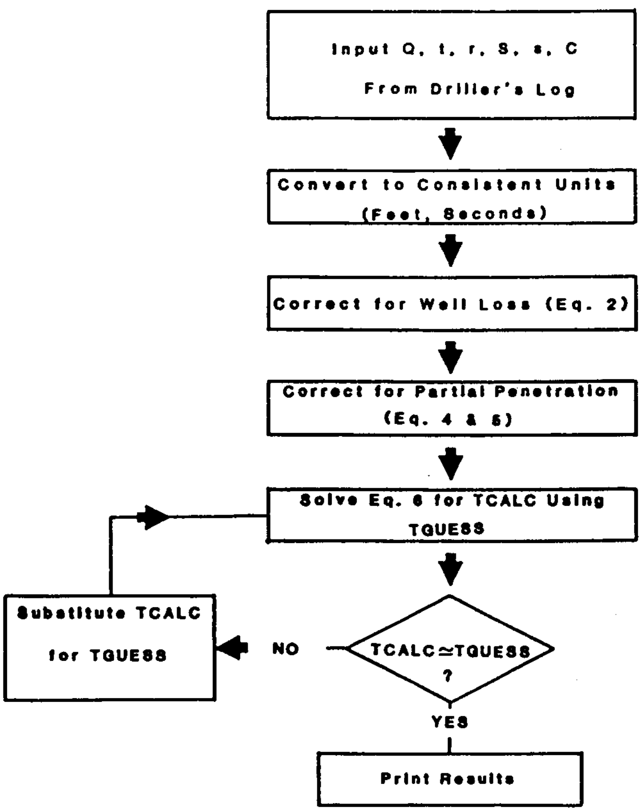

Fig. 2. Computer program flow chart.

Description of Field Sites

The aquifer analysis method described above was utilized for the two study areas in Wisconsin shown in Figure 1. The first (area A), called the Peninsula site, is in Door County, northeastern Wisconsin, and encompasses 17.8 mi² (46.1 km²). The aquifer at the Peninsula site is a highly fractured Silurian dolomite. Studies of the interactions of ground water at the site with surface water in adjacent Green Bay used computer modeling (Bradbury, 1982). The computer models required extensive data on transmissivity and hydraulic conductivity of the dolomite aquifer. Because the results of five available pumping tests in the area (Sherrill, 1978) might not adequately describe spatial variability of the fractured dolomite aquifer, the transmissivity estimation technique was applied to specific capacity data from 224 local wells. The use of specific capacity tests increased the average density of hydraulic conductivity data from 0.3 to 12.6 points/mi² (0.78 to 32.6 points/km²).

The second site (area B) encompasses a large portion of the Central Sand Plain of Wisconsin, which is underlain by an aquifer of sandy glacial outwash, and has an area of approximately 612 mi² (1585 km²). The sand and gravel aquifer in the

Table 1. Statistical Results of Estimates of Hydraulic Conductivity (K) from Specific Capacity for Two Areas in Wisconsin. Geometric Means, Standard Deviations (σ), and 95 Percent Confidence Limits Are Given

	K (ft/sec)
AREA A: Fractured dolomite (N = 223)	
Geometric mean	7.8×10^{-5}
σ	0.61
95% C.I.	$6.5 \times 10^{-5} - 9.3 \times 10^{-5}$
AREA B: Sandy outwash (N = 266)	
Geometric mean	2.1×10^{-3}
σ	0.25
95% C.I.	$1.6 \times 10^{-3} - 2.2 \times 10^{-3}$

area is widely utilized for spray irrigation of crops, especially potatoes. Recent indications of groundwater contamination by pesticides in the area (Rothschild *et al.*, 1982) prompted further study of the aquifer, including computer modeling (Rothschild, 1982). Specific capacity data for the area are abundant (268 points) in comparison to the number of pumping tests (11), and the transmissivity estimation technique was used to help describe the hydraulic characteristics of the aquifer. By utilizing specific capacity data the density of data points for transmissivity was increased from 0.018 points/mi^2 (pumping tests) to 0.44 points per mi^2 (0.045 to 1.14 points/km^2).

Results

Reliability of Estimates

Results of the computer estimation of hydraulic conductivities from specific capacity data agree well with values calculated using full-scale pumping tests. Table 1 gives a statistical summary of hydraulic conductivity estimates for 223 wells in fractured dolomite (area A) and 266 wells in sandy outwash (area B). Because hydraulic conductivity data are generally log-normally distributed (Freeze, 1975), the geometric mean gives a good measure of the central tendency of the data, and sigma (σ) represents the standard deviation of the log-transformed data. Table 1 shows that, using many data points, the specific capacity estimates give a lower mean hydraulic conductivity for fractured dolomite (7.8×10^{-5} ft/sec) than for sandy outwash (2.1×10^{-3} ft/sec). Standard deviation values show that the fractured dolomite has statistically more variation in hydraulic conductivity than does the sandy outwash, and that the range of variation in both materials is small enough to make the results useful. Freeze (1975) reports that computer models can give meaningful estimates of hydraulic head when hydraulic conductivity "σ of K" values are less than 0.5, but that meaningful head predictions are impossible when σ is greater than 2.0. Thus the σ values of 0.61 and 0.25 reported here give confidence of reasonable results when using the data in computer simulations to predict hydraulic heads.

In spite of the well-known difficulties in estimating hydraulic conductivities from specific capacity data, the range of values predicted by our method is relatively small. Figure 3 presents average hydraulic conductivities for various materials, and shows the range of values obtained from our computer estimates. As noted by Winter (1981) the standard error in estimating values of hydraulic conductivity is often close to 100 percent or even higher. Thus the ranges of values shown on Figure 3 are quite narrow when compared to the possible ranges of hydraulic conductivity values, and the variation in K is less than one order of magnitude for the sandy outwash and just over an order of magnitude for the fractured dolomite.

Comparing estimates from individual wells, the results of the computer program are surprisingly close to data determined by pumping tests (Table 2). In the fractured dolomite of area A (wells 1-5), specific capacity data give hydraulic conductivity estimates which are slightly smaller than but of the

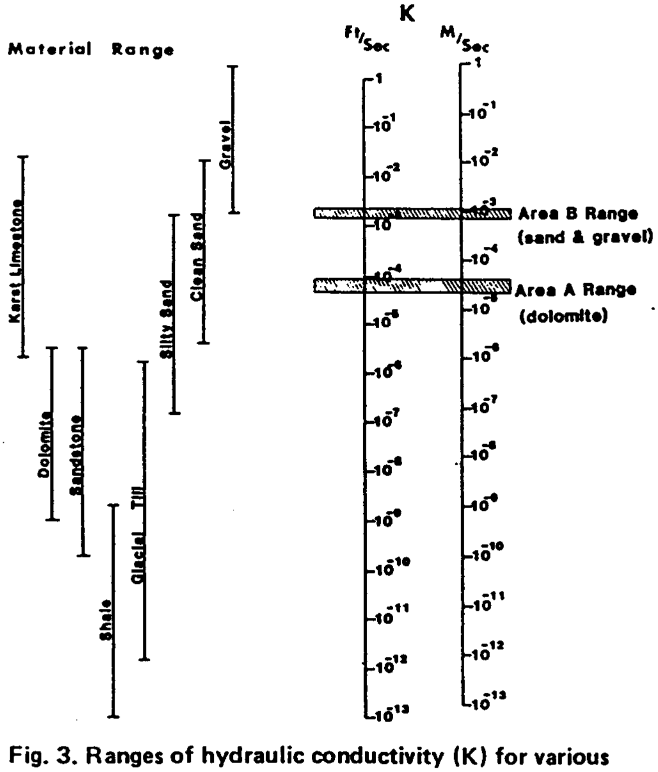

Fig. 3. Ranges of hydraulic conductivity (K) for various geologic materials, showing ranges determined from specific capacity estimates in this study (after Freeze and Cherry, 1979).

Table 2. Comparison of Values of Hydraulic Conductivity (K) Obtained from Pumping Tests with Values Estimated from Specific Capacities for Wells in Two Different Areas in Wisconsin

Well	*Pumping test K (ft/sec)*	*Specific capacity estimate K (ft/sec)*
AREA A: Fractured dolomite		
1	2.8×10^{-4}	7.3×10^{-4}
2	1.7×10^{-4}	1.0×10^{-5}
3	3.0×10^{-4}	5.0×10^{-4}
4	8.8×10^{-4}	2.8×10^{-4}
5	3.9×10^{-4}	1.0×10^{-4}
Geometric mean	3.5×10^{-4}	1.6×10^{-4}
σ	0.26	0.75
AREA B: Sandy outwash		
6	2.9×10^{-3}	1.5×10^{-3}
7	3.4×10^{-3}	1.5×10^{-3}
8	2.7×10^{-3}	2.8×10^{-3}
9	2.2×10^{-3}	1.8×10^{-3}
10	2.8×10^{-3}	1.8×10^{-3}
11	2.4×10^{-3}	2.0×10^{-3}
12	2.1×10^{-3}	1.8×10^{-3}
13	3.3×10^{-3}	2.7×10^{-3}
14	1.5×10^{-3}	1.9×10^{-3}
15	2.4×10^{-3}	2.2×10^{-3}
16	1.5×10^{-3}	2.8×10^{-3}
Geometric mean	2.4×10^{-3}	2.0×10^{-3}
σ	0.12	0.10

same order of magnitude as values derived from full-scale pumping tests using identical wells. In the sandy outwash of area B (wells 6-16), slight variations in K were also detected by specific capacity tests. Wells 9-12 in area B are radial collector wells. These wells are larger in diameter and are more efficient than the high capacity wells used for other specific capacity tests (Karnauskas, 1977). This efficiency difference is evident in consistently lower K values as determined by specific capacity tests, and highlights the importance of knowledge of well construction when interpreting such data. One of the poorer comparisons is for well 16. Due to the nature of outwash in this area the observation wells for the pumping test may not have been in full hydraulic connection with the pumping well. Much of the variation in values for the Central Sand Plain (area B) is explained by poor depth-to-bedrock control. Due to the high transmissivity of the overlying sands and gravels, few area wells are drilled to bedrock. In general, comparisons are poorer for the fractured dolomite of area A than for the sandy outwash of area B. The fractured dolomite is less homogeneous than the outwash, and the fracture system there may not truly approximate a porous media.

Contour Mapping

Contour maps of hydraulic conductivity for the two study areas are a valuable product of the computer program (Figures 4 and 5). The maps are produced by estimating T from specific capacity, then calculating K from aquifer thickness. Because all data are computerized, it is relatively simple to plot and contour the data using standard software packages. Interpolation, graphing, and smoothing packages were used to produce the maps in Figures 4 and 5 for the two study areas.

Distinct trends and differences are discernible in both areas. Figure 4 shows the hydraulic conductivity distribution in the fractured dolomite of the Peninsula area (area A). Because of the logarithmic distribution of K in the fractured dolomite the data are contoured by base 10 logs. As would be expected for a fractured dolomite aquifer, the areal distribution of K appears almost random with the exception of an area of higher K near the center of the area. The likelihood of this area having a higher K was confirmed by additional modeling efforts using a parameter estimation model (Bradbury, 1982).

In the sandy outwash of area B (Figure 5) the areal variation in K is less, and arithmetic contours are plotted. Variations in K shown on the map may be related to known depositional outwash facies in the area (Rothschild, 1982). The statistical inter-

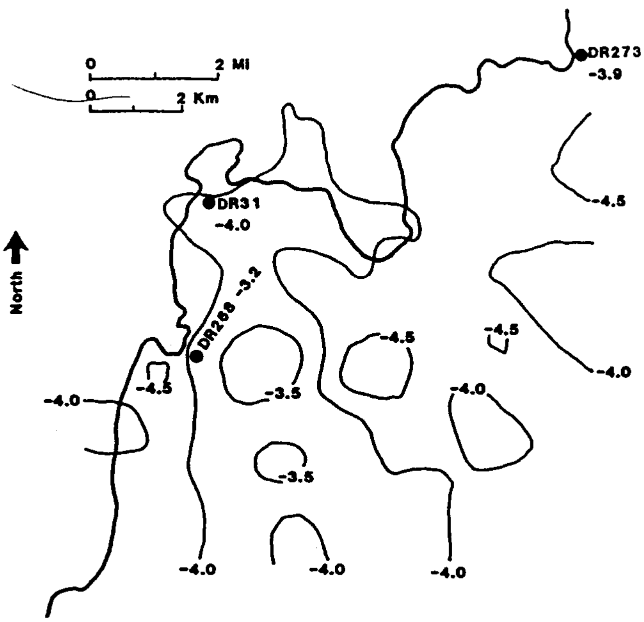

Fig. 4. Contour plot of hydraulic conductivity in study area A based on specific capacity and aquifer thickness data. Base 10 logs are plotted; contour interval is 0.5 log unit. Locations and log hydraulic conductivity values are shown for three wells where pumping tests were conducted.

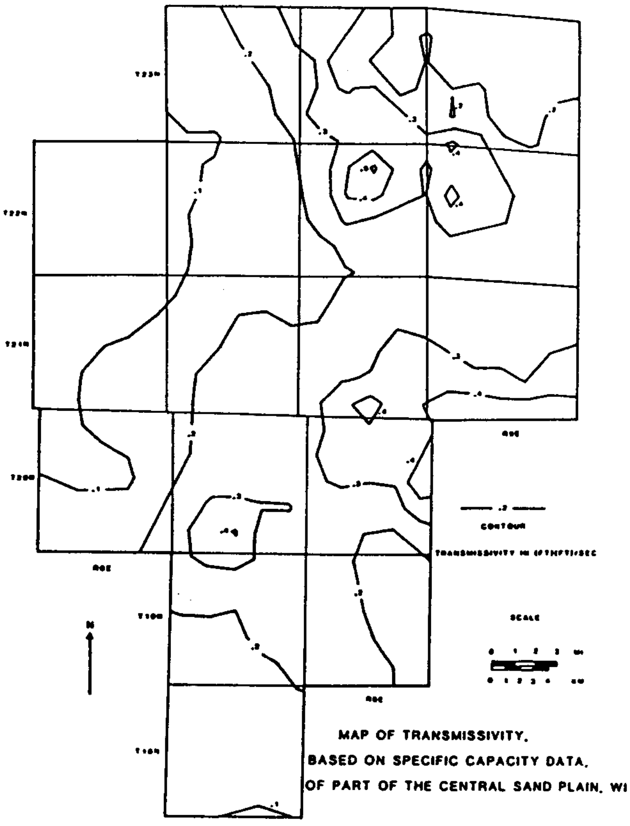

Fig. 5. Map of hydraulic conductivity based on specific capacity data for area B.

pretations of Figures 4 and 5 might be aided by advanced statistical techniques such as kriging which are beyond the scope of the present study.

Conclusions

Although the use of specific capacity data for estimating aquifer characteristics is not new, computer techniques can produce reliable estimates at more points and with less effort than in the past. Computers allow the rapid evaluation and manipulation of specific capacity data from large numbers of data points. The ability to use such data to describe the transmissivity and hydraulic conductivity of aquifers statistically or graphically is an important tool. The method described here has been successfully tested for sandy outwash and fractured dolomite aquifers at two field areas in Wisconsin.

Acknowledgments

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Appendix A

```
10  PRINT : PRINT : PRINT :
20  PRINT "A PROGRAM TO ESTIMATE AQUIFER TRANSMISSIVITY"
30  PRINT "AND HYDRAULIC CONDUCTIVITY"
40  PRINT "FROM SPECIFIC CAPACITY TESTS"
50  PRINT ""
60  PRINT "WRITTEN BY K. BRADBURY AND E. ROTHSCHILD. SEPTEMBER 1981"
70  PRINT ""
80  PRINT "**************************************************************"
90  PRINT "****************** LIST OF VARIABLES *****************"
100  PRINT
110  PRINT "NUM = IDENTIFICATION NUMBER OF WELL"
120  PRINT "DIAM = DIAMETER OF WELL (INCHES)"
130  PRINT "LGTH = LENGTH OF OPEN INTERVAL OR WELL SCREEN (FEET)"
140  PRINT "LVL = STATIC WATER LEVEL...NEGATIVE FOR FLOWING WELL (DEPTH I
     N FEET)"
150  PRINT "PUMP = DEPTH TO WATER WHILE PUMPING DURING SPECIFIC CAPACITY
     TEST (FEET)"
160  PRINT "LN = LENGTH OF TEST (HOURS)"
170  PRINT "GPM = PUMPING RATE DURING TEST (GALLONS/MINUTE)"
180  PRINT "AQTHIC = THICKNESS OF AQUIFER (FEET)"
190  PRINT "S = ESTIMATED OR MEASURED STORAGE COEFFICIENT (UNITLESS)"
200  PRINT "C = WELL LOSS COEFFICIENT (WALTON. BULL 49)... USE 1 IF UNKNO
     WN"
210  PRINT "SC = SPECIFIC CAPACITY CORRECTED FOR WELL LOSS (GALLONS/MINUT
     E/FOOT)"
220  PRINT "T = TRANSMISSIVITY (FEET * FEET/SECOND)"
230  PRINT "K = HYDRAULIC CONDUCTIVITY (FEET/SECOND)"
240  PRINT "ERR = CONVERGENCE CRITERIA FOR T ESTIMATE (FEET * FEET/SECOND
     )"
250  PRINT "***************************************************************
     "
260  PRINT "HOW MANY WELLS WILL BE ANALYZED?"
270  INPUT XX
280  DIM NUM(XX).DIAM(XX).LGTH(XX),LVL(XX),PUMP(XX),LN(XX).GPM(XX).AQTHIC
     (XX)
290  DIM SC(XX).S(XX).C(XX).T(XX),K(XX),KOUNT(XX),FLUB(XX).ITER(XX)
300 ERR = 0.1E - 5
310 KOUNT = 0
320 TGUESS = 0.1
330  REM  ***********************************************************
340  REM  READ IN RAW DATA IN UNITS GIVEN ON DRILLERS LOGS
350  REM  ***********************************************************
360  PRINT "DO YOU WANT TO ENTER DATA INTERACTIVELY OR FROM A FILE?"
370  PRINT "ENTER 0 IF INTERACTIVELY OR 1 IF FROM FILE"
380  INPUT A
390  IF A = 1 THEN  GOTO 530
400  FOR Z = 1 TO XX
410  PRINT "WELL NUMBER=": INPUT NUM(Z)
420  PRINT "WELL DIAMETER (IN)= ": INPUT DIAM(Z)
430  PRINT "STATIC WATER LEVEL (FT)= ": INPUT LVL(Z)
440  PRINT "DEPTH TO WATER DURING TEST (FT)= ": INPUT PUMP(Z)
450  PRINT "THE LENGTH OF THE TEST (HR)= ": INPUT LN(Z)
460  PRINT "PUMPING RATE (GPM)= ": INPUT GPM(Z)
470  PRINT "THICKNESS OF AQUIFER (FT)= ": INPUT AQTHIC(Z)
480  PRINT "OPEN INTERVAL (FT)= ": INPUT LGTH(Z)
490  PRINT "STORAGE COEFFICIENT= ": INPUT S(Z)
500  PRINT "WELL LOSS COEFFICIENT= ": INPUT C(Z)
510  NEXT Z
520  GOTO 560
530  FOR Z = 1 TO XX
540  READ NUM(Z).DIAM(Z).LVL(Z),PUMP(Z).LN(Z).GPM(Z).AQTHIC(Z).LGTH(Z).S(
     Z).C(Z)
550  NEXT Z
560  REM  ***********************************************************
570  REM  DO ANALYSIS FOR EACH WELL
580  REM  ***********************************************************
590  FOR Y = 1 TO XX
600 FLUB(Y) = 0:ITER(Y) = 0
610  REM  ***********************************************************
620  REM  CHANGE TO CONSISTENT UNITS AND CALCULATE DRAWDOWN
630  REM  ***********************************************************
640 R = DIAM(Y) / 24.0
650 TIME = LN(Y) * 3600.0
660 Q = GPM(Y) / 449.0  <---- Q converted from gpm to ft^3/sec
670 DD =  - (LVL(Y) - PUMP(Y))
680  IF (DD <  = 0.0) GOTO 1090
690 KOUNT = KOUNT + 1
700  REM  ***********************************************************
710  REM  CORRECT DRAWDOWN FOR WELL LOSS USING THE EQUATION SW=CQQ
720  REM  SEE WALTON. BULL. 49. PAGE 27
730  REM  C IS ESTIMATED FROM STEP DRAWDOWN TESTS.
740  REM  ***********************************************************
750 SW = C(Y) * Q * Q
760 DD = DD - SW
770 SC(Y) = GPM(Y) / DD
780  REM  ***********************************************************
790  REM  CALCULATE AQUIFER TRANSMISSIVITY USING THE JACOB EQUATION
800  REM  USING A CORRECTION FOR PARTIAL PENETRATION AS GIVEN BY
810  REM  STERNBURG (1973)
820  REM  ***********************************************************
830  REM  FIRST CALCULATE SP PARAMETERS FOR USE IN THE EQUATION
840  REM  ***********************************************************
850 B = LGTH(Y) / AQTHIC(Y)
860  IF (B > 1.0) GOTO 1090
870 HRW = AQTHIC(Y) / R
880 GB = 2.9480 - (7.363 * B) + (11.447 * B * B) - (4.675 * B * B * B)
890 SP = ((1.0 - B) / B) * ( LOG (HRW) - GB)
900  REM  ***********************************************************
910  REM  NOW SOLVE FOR T USING ITERATIONS
920  REM  ***********************************************************
930 TGUESS = 0.1
940  FOR W = 1 TO 25
950 F1 = Q / (4.0 * 3.1416 * DD)
960 F2 = (2.25 * TGUESS * TIME) / (R * R * S(Y))
970 TCALC = F1 * ( LOG (F2) + (2.0 * SP))
980 TEST =  ABS (TCALC - TGUESS)
990 TGUESS =  ABS (TCALC)
1000  IF (TEST    = ERR) THEN  GOTO 1060
1010 KOUNT(Y) = W
1020  NEXT W
1030  IF (KOUNT(Y) = 25) AND (TEST   ERR) THEN  GOTO 1050
1040  GOTO 1060
1050 ITER(Y) = 1: GOTO 1100
1060 T(Y) = TCALC
1070 K(Y) = T(Y) / AQTHIC(Y)
1080  GOTO 1100
1090 FLUB(Y) = 1
1100  NEXT Y
1110  REM  ***********************************************************
1120  REM  PRINT OUTPUT
1130  REM  ***********************************************************
1140  PR# 1
1150  PRINT "***********************************************************"
1160  PRINT "AQUIFER PROPERTIES AS DETERMINED BY ANALYSIS OF "
1170  PRINT "              SPECIFIC CAPACITY TESTS"
1180  PRINT "***********************************************************"
1190  PRINT ""
1200  FOR V = 1 TO XX
1210  IF FLUB(V) = 1 GOTO 1330
1220  IF ITER(V) = 1 GOTO 1390
1230  PRINT ""
1240  PRINT "WELL NUMBER ";NUM(V)
1250  PRINT "SPECIFIC CAPACITY (GPM/FT) = ";SC(V)
1260  PRINT "TRANSMISSIVITY (FT*FT/SEC) = ";T(V)
1270  PRINT "              USING A STORAGE COEFFICIENT = ";S(V)
1280  PRINT "              NUMBER OF ITERATIONS = ";KOUNT(V)
1290  PRINT "HYDRAULIC CONDUCTIVITY (FT/SEC) = ";K(V)
1300  NEXT V
1310  PRINT "THE NUMBER OF WELLS IN THIS RECORD IS ";XX
1320  GOTO 1430
1330  PRINT ""
1340  PRINT "WELL NUMBER ";NUM(V)
1350  PRINT "INPUT ERROR. EITHER:"
1360  PRINT "      1. WATER LEVEL WAS HIGHER DURING TEST THAN BEFORE. OR:"

1370  PRINT "      2. THE SCREEN LENGTH IS LONGER THAN THE AQUIFER THICKNE
     SS."
1380  GOTO 1300
1390  PRINT ""
1400  PRINT "WELL NUMBER ";NUM(V)
1410  PRINT "SOLUTION DID NOT CONVERGE WITHIN 25 ITERATIONS"
1420  GOTO 1300
1430  END
```

Appendix B

As an example of computer program input and output, the following data from area A were input into the interactive computer program (Appendix A).

Number of wells to be analyzed = 2
Interactive data entry

Well number 1
Well diameter = 6 in.
Static water level = 42 ft
Depth to water during test = 57 ft
Length of test = 8 hr
Pumping rate = 10 gpm
Aquifer thickness = 205 ft
Open interval = 47 ft
Storage coefficient = 0.0002
Well loss coefficient = 32.7 sec^2/ft^5 (high!?)

Well number 2
Well diameter = 6 in.
Static water level = 132 ft
Depth to water during test = 141 ft
Length of test = 8 hr
Pumping rate = 10 gpm
Aquifer thickness = 115 ft
Open interval = 68 ft
Storage coefficient = 0.0002
Well loss coefficient = 32.7 sec^2/ft^5

Figure A1 is the computer output generated by these data.

```
******************************************************************
AQUIFER PROPERTIES AS DETERMINED BY ANALYSIS OF
           SPECIFIC CAPACITY TESTS
******************************************************************
WELL NUMBER 1
SPECIFIC CAPACITY (GPM/FT) = .666688713
TRANSMISSIVITY (FT*FT/SEC) = 5.93317103E-03
           USING A STORAGE COEFFICIENT = 2E-04
           NUMBER OF ITERATIONS = 3
HYDRAULIC CONDUCTIVITY (FT/SEC) = 2.89422977E-05

WELL NUMBER 2
SPECIFIC CAPACITY (GPM/FT) = 1.11117235
TRANSMISSIVITY (FT*FT/SEC) = 4.56944391E-03
           USING A STORAGE COEFFICIENT = 2E-04
           NUMBER OF ITERATIONS = 3
HYDRAULIC CONDUCTIVITY (FT/SEC) = 3.97342949E-05
THE NUMBER OF WELLS IN THIS RECORD IS 2
```

Fig. A-1. Example of computer printout.

Jacob 1946: ASCE "Proceedings (Papers), pp. 629-646
Jacob 1947: ASCE "Transactions", Paper no. 2321, pp. 1047-1064 [discussions pp.1065-1070]

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TRANSACTIONS

Paper No. 2321

DRAWDOWN TEST TO DETERMINE EFFECTIVE RADIUS OF ARTESIAN WELL

By C. E. Jacob,[1] Assoc. M. ASCE

With Discussion by Messrs. N. S. Boulton, Carl Rohwer, R. M. Leggette, M. R. Lewis, and C. E. Jacob.

Synopsis

The drawdown in an artesian well that is pumped has two components: The first, arising from the "resistance" of the formation, is proportional to the discharge; and the second, termed "well loss" and representing the loss of head that accompanies the flow through the screen and upward inside the casing to the pump intake, is proportional approximately to the square of the discharge. The resistance of an extensive artesian bed increases with time as the ever-widening area of influence of the well expands. Consequently, the specific capacity of the well, which is discharge per unit drawdown, decreases both with time and with discharge.

The multiple-step drawdown test outlined in this paper permits the determination of the well loss and of the "effective radius" of the well. The trend of drawdown is observed in the pumped well and in one or more near-by observation wells as the discharge is increased in stepwise fashion. A simple graphical procedure gives the permeability and the compressibility of the bed. From these several factors it is possible to predict the pumping level at any time for any given discharge.

Notation

The letter symbols in this paper are defined where they first appear and are assembled alphabetically, for convenience of reference, in the Appendix.

Introduction

It has long been known that the discharge of an artesian well is almost, but not quite, proportional to the drawdown. In a well that is pumped the drawdown is the difference between the static water level and the pumping

Note.—Published in May, 1946, *Proceedings*. Positions and titles given are those in effect when paper or discussion was received for publication.

[1] Hydr. Engr., Water Resources Branch, U. S. Geological Survey, Washington, D. C.

water lev.., customarily measured after several hours of continuous operation. Usually the major part of this loss of head, or drawdown, occurs in the formation, where the energy expended in overcoming the frictional resistance of the sand against the slowly moving water is directly proportional to that rate of motion. A smaller although no less important part of the loss of head occurs as the water moves at relatively high velocities through the screen and upward inside the casing to the intake of the pump. This head loss is approximately proportional to some higher power of the velocity approaching the square of the velocity. Adding these two components of drawdown:

$$s_w = B\,Q + C\,Q^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

—approximately. Considering the drawdown, s_w, analogous to electric potential drop and the discharge, Q, analogous to electric current, the factor B can be defined as the "resistance" of the formation. This factor represents the total hydraulic resistance of the formation, from the face of the well to some distance where the head drop is virtually zero and where the radial motion of water toward the discharging well has not yet begun. The ratio of discharge to drawdown, called "specific capacity," is seen from Eq. 1 to be

$$Q/s_w = 1/(B + C\,Q) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Clearly, then, the specific capacity must vary, however slightly, with the discharge. Also, it must vary with time because, as will be shown, the resistance B increases with time as the ever-widening area of influence of the well expands.

It is the purpose of this paper to demonstrate that the factors B and C can be determined by a procedure that is little more elaborate than the usual "drawdown test" made to determine the specific capacity and to check the performance of the pump and motor. This is accomplished simply by controlling, more closely, the stepwise variation of the discharge and by observing, more frequently and more accurately, the trend of the pumping level as it is lowered.

Distribution of Drawdown in and Near an Artesian Well

Fig. 1 shows three typical examples of artesian wells that completely penetrate extensive formations of assumedly homogeneous structure and uniform thickness. Fig. 1(a) is the ideal case of an uncased hole drilled through a water-bearing sandstone confined above and below by impervious shales. Virtually all the head loss occurs in the formation, since no well screen or slotted casing is present to impede the flow of water into the hole There must inevitably be some additional friction as the water moves up the hole, and consequently the pumping water level in the hole does not coincide exactly with the head at the face of the hole just inside the formation but rather stands somewhat lower, perhaps as indicated by the dashed line in Fig. 1(a).

Fig. 1(b) shows a common type of construction in an unconsolidated artesian sand, where a screen is necessary. For comparison, this sand is shown as having the same thickness and permeability as the sandstone of Fig. 1(a)

In the second case, with the same discharge there is the same drop in head within the formation; but in addition there is a "well loss" which includes the head lost through friction as the water flows upward inside the screen and casing to the pump intake as well as to the head drop across the screen. The radial distribution of head within the formation is about the same in both

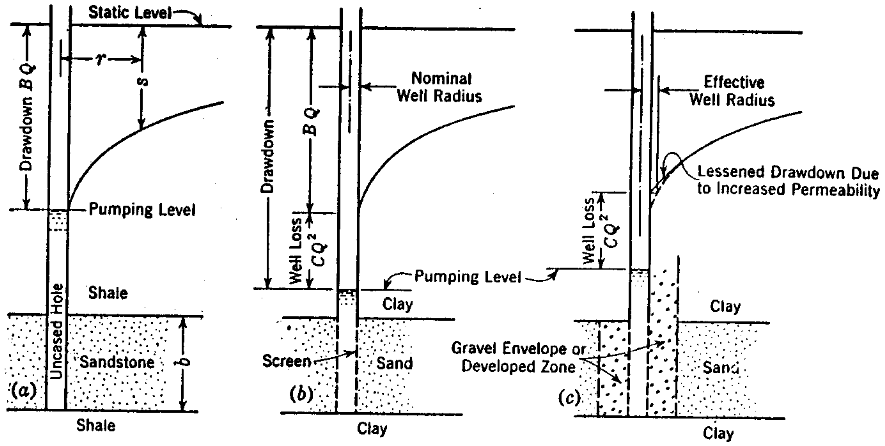


Fig. 1.—Typical Examples of Artesian Wells, Showing Distribution of Drawdown

cases, which may indicate that the sand in Fig. 1(b) is not greatly affected by drilling or developing operations; the screen effectively retains all the sand including fines.

A common, although not always essential, item of artesian well construction in unconsolidated formations—the gravel envelope—is shown in Fig. 1(c). The gravel envelope is particularly effective when the water-bearing sand is fine and of uniform grading. When properly constructed, it is also useful in other situations—to prevent the fines from being drawn into the well. If the size of gravel is properly chosen, the head loss in the immediate vicinity of the screen is reduced to less than it would be if the natural undisturbed water-bearing formation which the gravel replaced were there. Developing a well to remove the fines from the material surrounding the screen has a similar effect. In some sands, developing operations alone are adequate and gravel-wall construction is not needed. In either case the increased permeability of the material surrounding the well lessens the drawdown and increases the effective radius of the well. "Effective radius" is defined as that distance, measured radially from the axis of the well, at which the theoretical drawdown based on the logarithmic head distribution (defined subsequently by Eq. 4) equals the actual drawdown just outside the screen (see Fig. 1(c)).

The dashed curved line in Fig. 1(c) represents the head distribution that would exist if the water-producing bed were left in place undisturbed, with uniform permeability. That curve duplicates the drawdown curves in Figs. 1(a) and 1(b). It is so shaped because, to maintain a steady flow of water (at the rate Q) toward the well, the hydraulic gradient must be inversely proporti[illegible]l to the radial distance; or, actually:

$$\frac{ds}{dr} = -\frac{Q}{2\pi k b r} \qquad (3)$$

in which b is the thickness of the bed and k is the "permeability" or transmission constant of the sand, defined as:

> "* * * the quantity [volume] of water that would be transmitted in unit time through a cylinder of the soil of unit length and unit cross-section under unit difference in head at the ends."[2]

Integrating Eq. 3 between the fixed limit r_w and the variable limit r:

$$s_w - s = \frac{Q}{2\pi T} \log_e \frac{r}{r_w} \qquad (4)$$

In Eq. 4, the "transmissibility," T, is the product of k and b.[3] Eq. 4 gives a logarithmic distribution of drawdown that holds in the immediate vicinity of a well pumping from an artesian bed which it penetrates completely. Assuming that the drawdown, s_w, is known at the effective well radius, r_w, the drawdown at some greater distance may be determined easily. Actually, as a matter of common knowledge, the drawdown in an artesian well increases continuously with time (rapidly at first, of course, and then more slowly) as long as the discharge continues at a steady rate and also provided that the well is not too near the margin of the aquifer where the head may be maintained essentially constant despite the withdrawal of water. To determine the drawdown at the well and its distribution throughout an extensive aquifer at any time, it is necessary to study the flow of the confined water in response to varying heads more closely, taking into consideration the compressibility of the water and also the compressibility of the sand bed.

Theory of Nonsteady Radial Flow in an Extensive Artesian Aquifer

Consider a cylindrical shell of height b, inner radius r, and outer radius $(r + \delta r)$ concentric with the axis of the well. By the principle of continuity, the net outward flow of water from this shell must equal the time rate of decrease of the volume of water within the shell, referred to a constant (atmospheric) pressure. The total volume of water in the shell is

$$V_w = 2\pi r\, \delta r\, b\, n \qquad (5)$$

in which n is the porosity of the sand. The time rate of decrease of this volume is $2\pi r\, \delta r\, b\, n\, \beta \frac{\partial(\gamma s)}{\partial t}$, in which γ is the specific weight of the water and β is its compressibility. To allow for the compressibility of the water-producing bed, which is assumed to be compacted elastically as the pressure is reduced

[2] "Theoretical Investigation of the Motion of Ground Waters," by Charles S. Slichter, *Nineteenth Annual Report*, U. S. Geological Survey, 1899, Pt. II, p. 323.

[3] "The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground-Water Storage," by Charles V. Theis, *Transactions*, Am. Geophysical Union, Pt. II, 1935, p. 520.

and as the water is allowed to expand, an apparent compressibility, β', is substituted for β. Experience[4,5] indicates β' to be several times the actual water compressibility, β. Combining several factors into a nondimensional coefficient,

$$S = \gamma\, \beta'\, b\, n \qquad (6a)$$

and $2\pi r\, \delta r\, S \frac{\partial s}{\partial t}$ is the time rate of decrease of the volume of water. In Eq. 6a S is the "coefficient of storage,"[6,7] which defines the volume of water that a unit decline in head releases from storage in a vertical prism of the aquifer of unit cross-sectional area.

The apparent fluid compressibility, β', is related to the respective compressibilities of the water and the sand as follows:

$$\beta' = \beta + \frac{\alpha}{n} \qquad (6b)$$

Again β is the compressibility of the water, and n is the porosity of the sand. The symbol α represents the "compressibility" of the sand bed—that is, the relative decrease in thickness of the bed per unit increase of the vertical component of compressive stress in the sand.

The foregoing relation in somewhat different notation was derived in an earlier paper[8] by the writer in which the theory of nonsteady flow in elastic artesian aquifers was developed.

The outward flow of water from the shell through its inner cylindrical surface is equal to $-2\pi r T\left(\frac{\partial s}{\partial r}\right)$. Similarly, the outward flow through the outer cylindrical surface is $2\pi T (r + \delta r)\left(\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}\delta r\right)$. Assuming the upper and lower bounding planes of the aquifer to be impermeable, the sum of these two terms may be equated to the time rate of decrease of the enclosed volume of water. Expanding, eliminating differentials of higher order, simplifying, and dividing through by $(2\pi r T \delta r)$:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t} \qquad (7)$$

The solution of this fundamental differential equation that is sought here must satisfy the following conditions:

$$s = 0 \quad \text{for} \quad t \leqq 0 \qquad (8a)$$

$$\lim_{r \to \infty} s = 0 \quad \text{for} \quad t > 0 \qquad (8b)$$

[4] "Notes on the Elasticity of the Lloyd Sand on Long Island, New York," by C. E. Jacob, *Transactions*, Am. Geophysical Union, 1941, Pt. III, pp. 783–787.

[5] "Application of Coefficients of Transmissibility and Storage to Regional Problems in the Houston District, Texas," by W. F. Guyton, *ibid.*, pp. 756–770.

[6] "The Significance and Nature of the Cone of Depression in Ground-Water Bodies," by Charles V. Theis, *Economic Geology*, 1938, p. 894.

[7] "The Source of Water Derived from Wells," by Charles V. Theis, *Civil Engineering*, May, 1940, p. 277.

[8] "On the Flow of Water in an Elastic Artesian Aquifer," by C. E. Jacob, *Transactions*, Am. Geophysical Union, 1940, Pt. II, pp. 574–586.

and

$$\underset{r \to 0}{\text{Limit}} \left(r \frac{\partial s}{\partial r} \right) = - \frac{Q}{2 \pi T} \qquad \text{for} \qquad t > 0 \ldots\ldots\ldots\ldots (8c)$$

The answer is given in terms of an infinite series[8,9,10] as follows:

$$s = \frac{Q}{4 \pi T} \left(- 0.5772 - \log_e u + u - \frac{u^2}{2 \times 2!} - \frac{u^3}{3 \times 3!} + \cdots \right) \ldots (9)$$

in which

$$u = \frac{t^*}{t} = \frac{r^2 S}{4 T t}, \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

In still simpler notation,

$$s = \frac{Q}{4 \pi T} W(u) \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Fig. 2 gives a nondimensional plotting of Eq. 9 or of Eq. 11. The well starts pumping at a steady rate Q at time zero ($t/t^* = 0$). The drawdown at a given distance from the well increases very slowly at first and reaches a

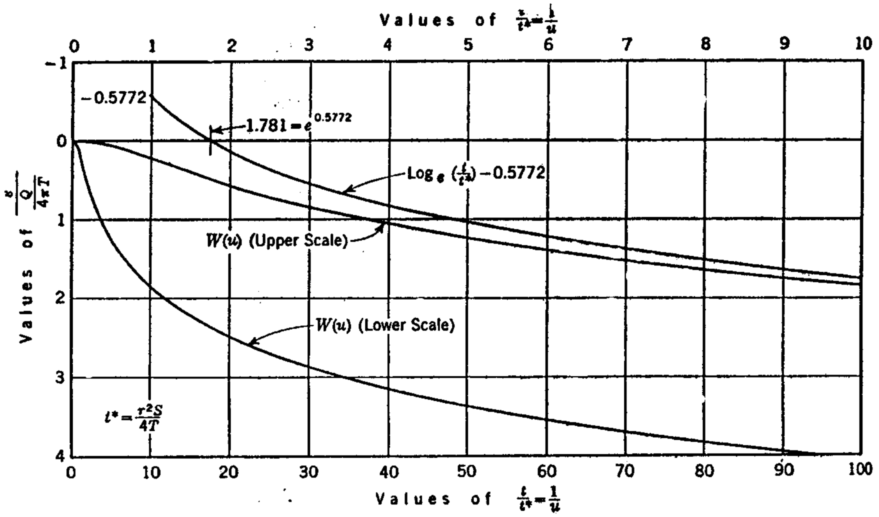


FIG. 2.—NONDIMENSIONAL TIME-DRAWDOWN CURVES, EXACT AND APPROXIMATE, FOR SINGLE WELL DISCHARGE AT A STEADY RATE, FROM AN EXTENSIVE ARTESIAN AQUIFER

maximum rate of increase at $t/t^* = 1$. As this is the point of inflection on the time-drawdown curve, t^* may be called the "inflectional time." Thereafter the rate of increase of drawdown diminishes continually but never vanishes. Theoretically, the drawdown becomes infinite at infinite time.

In Fig. 3 the same equation is plotted on semilogarithmic paper, again in nondimensional form. For sufficiently large values of t, the W-function may be approximated by a simple logarithmic expression that plots as a straight

[9] "The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground-Water Storage," by Charles V. Theis, *Transactions*, Am. Geophysical Union, 1935, Pt. II, pp. 519–524.

[10] "The Flow of Homogeneous Fluids Through Porous Media," by M. Muskat, McGraw-Hill Book Co. Inc., New York, N. Y., 1937, p. 667.

line on that graph. Thus, the drawdown after sufficient time has elapsed is given approximately by

$$s = \frac{Q}{4 \pi T} \left(\log_e \frac{t}{t^*} - 0.5772 \right) \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

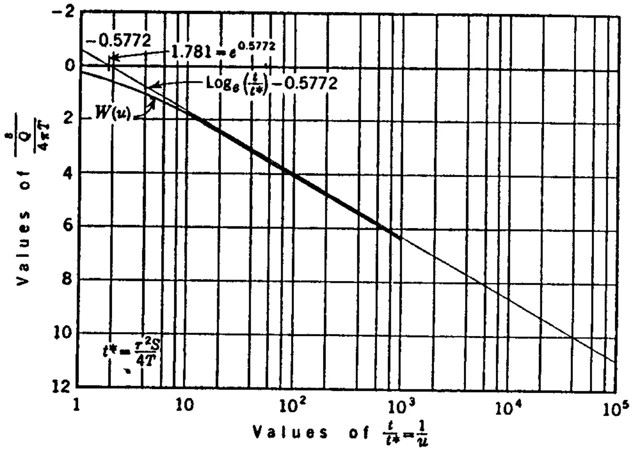


FIG. 3.—SEMILOGARITHMIC PLOTTING OF THEORETICAL TIME-DRAWDOWN CURVE AND STRAIGHT-LINE APPROXIMATION

The drawdown in a well with a negligible well loss (Fig. 1(*a*)) is then:

$$s_w = \frac{Q}{4 \pi T} \left(\log_e \frac{t}{t^*_w} - 0.5772 \right) \ldots\ldots\ldots\ldots (13a)$$

in which $t^*_w = r^2_w \frac{S}{4T}$, r_w being the effective radius of the well. When the well loss is appreciable (Figs. 1(*b*) or 1(*c*)), the drawdown in the well is

$$s_w = \frac{Q}{4 \pi T} \left(\log_e \frac{t}{t^*_w} - 0.5772 \right) + C Q^2 \ldots\ldots\ldots (13b)$$

Comparing Eq. 13*b* with Eq. 1, the resistance of the aquifer is

$$B = \frac{\log_e \frac{t}{t^*_w} - 0.5772}{4 \pi T} \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

According to Eq. 12 or Eq. 13*a*, S and T may be determined from a series of drawdown observations by plotting values of s against values of the logarithm of t. For sufficiently large values of t, relative to t^* or t^*_w, the points should fall on a straight line. Taking two points on that line to determine the slope,

$$T = \frac{2.30\, Q \log_{10} \frac{t_2}{t_1}}{4 \pi (s_2 - s_1)} \ldots\ldots\ldots\ldots\ldots\ldots (15a)$$

Eq. 15*a* can be simplified further by choosing, arbitrarily the two points one log cycle apart. Then, $\log_{10} \frac{t_2}{t_1} = 1$, and

$$T = \frac{2.30\, Q}{4\,\pi\,(s_2 - s_1)} \qquad (15b)$$

Knowing T, theoretically the value of S may now be determined from the intercept of the straight line with the zero-drawdown line because at this point $(t, 0)$—

$$\log_e \frac{4\,T\,t}{r^2\,S} = 0.5772 \qquad (16)$$

from which

$$S = \frac{4\,T\,t}{r^2\,e^{0.5772}} = \frac{2.25\,T\,t}{r^2} \qquad (17)$$

Application of Theory to a Simple Drawdown Test

Fig. 4 is a semilogarithmic graph of data from a simple drawdown test of a pumping well and a near-by observation well in glacial outwash near Meadville, Pa. The pumping well is of the gravel-wall type and has 15 ft of 18-in. screen between depths of 49 ft and 64 ft. During the test it was pumped at Q = 1,350 gal per min, or about 3.0 cu ft per sec. Observations of drawdown were made periodically by an air line in the pumping well. An automatic gage gave a continuous record of the drawdown and subsequent recovery in an observation well 1,200 ft away from the pumping well. The data for the drawdown period are plotted as open circles. The data for the recovery period are plotted as solid circles.

The transmissibility may be determined from the slope of either straight line. In both cases the change in drawdown over one log cycle is 2.27 ft. According to Eq. 15*b*, with Q equal to 3.0 cu ft per sec, the value of T is $\frac{2.30 \times 3.0}{4\,\pi \times 2.27} = 0.24$ sq ft per sec. The storage coefficient may be determined from the intercept of the upper straight line with the zero-drawdown line by substituting t = 693 sec in Eq. 17. The result of this calculation is $S = \frac{2.25 \times 0.24 \times 693}{1.44 \times 10^6} = 0.00026$.

Assuming the porosity of the sand to be 35% and its effective thickness about 100 ft, the apparent water compressibility is computed as follows: By Eq. 6*a*,

$$\beta' = \frac{S}{\gamma\,n\,b} = \frac{0.00026}{62.4 \text{ (lb per cu ft)} \times 0.35 \times 100 \text{ (ft)}}$$

$$= \frac{1}{8{,}400{,}000 \text{ (lb per cu ft)}} = \frac{1}{58{,}000 \text{ (lb per cu in.)}}$$

[4] The bulk modulus of gas-free water at ordinary temperatures is about 300,000 lb per sq in. The apparent water compressibility (which is the reciprocal of bulk modulus) in this case is then about five times the actual compressibility of water.

Assuming that this value of S holds within the immediate vicinity of the well, the well loss and the factor C may be determined under the further provisional assumption that the effective well radius is equal to the nominal well radius (see Fig. 1(*b*)).

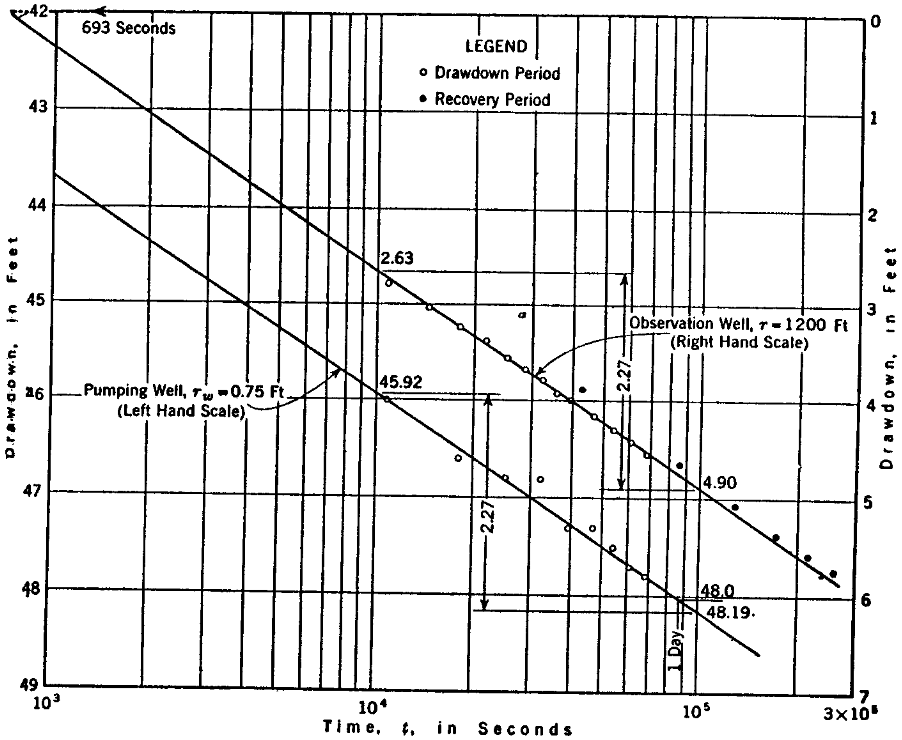


Fig. 4.—Semilogarithmic Plotting of Data from a Simple Drawdown Test of a Pumping Well and a Near-By Observation Well to Determine Transmissibility, Storage Coefficient, and Well Loss

For example, to determine the drawdown for one day, with the use of Fig. 4, solve the formula:

$$s_w = \frac{Q}{4\,\pi\,T}\left(2.303 \log_{10} \frac{4\,T\,t}{r_w^2\,S} - 0.5772\right) + C\,Q^2 \qquad (18)$$

The fraction $\frac{4\,T\,t}{r_w^2\,S} = \frac{4 \times 0.24 \times 86{,}400}{0.5625 \times 2.6 \times 10^{-4}} = 5.67 \times 10^8$; $\log_{10} \frac{4\,T\,t}{r_w^2\,S} = 8.753$; the value in parentheses in Eq. 18 equals $(8.753 \times 2.303) - 0.58 = 19.5$; and $s_w = \frac{3.0 \times 19.58}{4\,\pi \times 0.24} + C\,Q^2 = 19.5 + C\,Q^2 = 48.0$ ft. Finally, $C\,Q^2 = 9.0\,C = 48.0 - 19.5 = 28.5$ ft; from which $C = 3.2 \left(\frac{\sec^2}{\text{ft}^5}\right)$. The foregoing calculations indicate that $B\,Q$ in this case was about 19.5 ft after 24 hours of continuous pumping. The observed drawdown in the pumping well at that time was 48.0 ft, leaving .5 ft for the well loss. Inasmuch as Q was about 3.0 cu ft per sec, the numerical value of C was approximately 3.2.

Fig. 5 shows how the specific capacity of the well under discussion would vary with the discharge or with time. Because of the relatively high well loss, the one-day specific capacity for $Q = 3$ cu ft per sec is only about 60% of that for $Q = 1$ cu ft per sec. The one-year specific capacity for 3 cu ft per sec

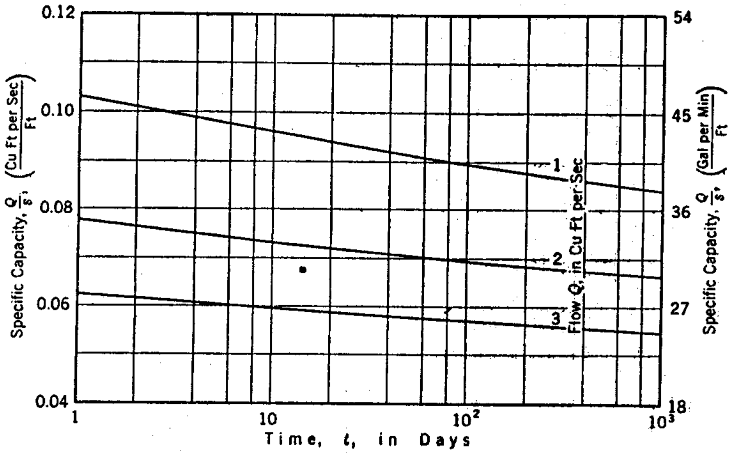


FIG. 5.—VARIATION OF THE SPECIFIC CAPACITY OF PUMPING WELL OF FIG. 4 WITH DISCHARGE AND WITH TIME

is about 65% of that for 1 cu ft per sec. Since at lower discharge rates a greater proportion of the total drawdown is attributable to head loss occurring within the formation (which increases with time while the other component remains constant), the specific capacity shows a greater percentage decline at the lower discharge rates. This fact is shown clearly in Fig. 5. With $Q = 1$ cu ft per sec, the specific capacity declines from about 0.104 cu ft per sec per ft at one day to about 0.086 cu ft per sec per ft at one year—a drop of about 17%. On the other hand, with $Q = 3$ cu ft per sec, the specific capacity declines from about 0.063 cu ft per sec per ft at one day to about 0.056 cu ft per sec per ft at one year—a drop of about 11%.

Fig. 5 illustrates the importance of stating the length of the pumping period during which the discharge remains constant and at the end of which the reported specific capacity is to be determined. It is also important to state the discharge. Too often in the past the specific capacity has been regarded as invariable, only passing attention being given to its variation with discharge and little or nothing being noted about its variation with time. This neglect may perhaps be attributed to the fact that, most commonly measurements of drawdown are made by an air line, with a pressure gage reading to the nearest pound per square inch or an altitude gage reading to the nearest foot no effort being made to interpolate closer than half a scale division. Very often the discharge is allowed to vary unmethodically to obtain several points quickly on the discharge-head curve for checking the characteristics of the pump and motor.

In the example given in this section it was assumed that the gravel envelope was not particularly effective for reducing the drawdown in the vicinity of the well and that therefore the nominal radius of the well screen might be used for the effective radius of the well. Actually, a more practical line of attack is to assume that conditions are as indicated in Fig. 1(*c*) and then to devise a method of determining the well loss that is independent of the effective well radius, which is to be determined last. The theory of such a method is outlined in the following section.

Theory of Multiple-Step Drawdown Test to Determine Well Loss and Effective Well Radius

Eq. 13*b* gives the drawdown in an artesian well with an appreciable well loss. It applies to a single drawdown period preceded by a period during which the well is idle. By modifying the second term of the right-hand member, Eq. 13*b* may be made to apply to increments of drawdown occurring in successive periods, at the beginning of each of which the discharge is increased abruptly.

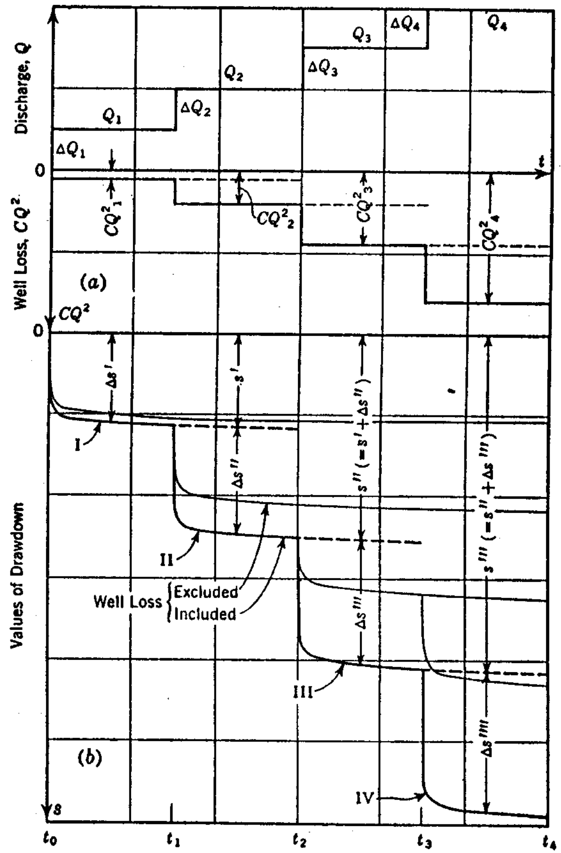


FIG. 6.—VARIATION OF DISCHARGE, DRAWDOWN, AND WELL LOSS IN MULTIPLE-STEP DRAWDOWN TEST

Fig. 6 depicts the progressive lowering of head in a multiple-step drawdown test. Two sets of drawdown curves are shown: The light lines in Fig. 6(*b*) show the draw-down that would occur during the successive periods of the test if there were no well loss; and the heavy lines, to which the notations refer, include the well losses, $C\,Q^2$, values of which are indicated in Fig. 6(*a*).

At time $t = t_0$ if the well (which theretofore has been idle) is started pumping at a rate $Q_1 = \Delta Q_1$, the drawdown at any time t thereafter is given by

$$\Delta s' = \frac{\Delta Q_1}{4\,\pi\,T}\left(\log_e \frac{t'}{t^*_w} - 0.5772\right) + C\,Q_1^2 \qquad (19a)$$

in which $t' = t - t_0$. If at some later time t_1 the discharge is increased by an amount ΔQ_2 to a new rate Q_2, the second increment of drawdown obeys the relation,

$$\Delta s'' = \frac{\Delta Q_2}{4 \pi T}\left(\log_e \frac{t''}{t^*_w} - 0.5772\right) + C\,(Q^2_2 - Q^2_1) \quad \ldots\ldots (19b)$$

with $t'' = t - t_1$.

Similarly, the third increment of drawdown beginning at time t_2 obeys the relation,

$$\Delta s''' = \frac{\Delta Q_3}{4 \pi T}\left(\log_e \frac{t'''}{t^*_w} - 0.5772\right) + C\,(Q^2_3 - Q^2_2) \quad \ldots\ldots (19c)$$

in which $t''' = t - t_2$. The generalized equation of this progression is

$$\Delta s^{(i)} = \frac{\Delta Q_i}{4 \pi T}\left(\log_e \frac{t^{(i)}}{t^*_w} - 0.5772\right) + C\,(Q^2_i - Q^2_{i-1}) \quad \ldots\ldots (20)$$

in which $t^{(i)} = t - t_{i-1}$.

The value of T might be determined from any one of these equations by plotting $\Delta s^{(i)}$ against $\log_e t^{(i)}$, as before. However, to determine $r^2_w S$, from which to solve for r_w, the constant C must first be found, as this factor shifts the intercept according to the magnitude of Q_{i-1} and Q_i.

Dividing Eqs. 19 and 20 by the respective increments of discharge, and fixing $t' = t'' = t''' \cdots = t^{(i)}$, after simplifying the differences in squares of values of discharge:

$$\left.\begin{aligned} \frac{\Delta s'}{\Delta Q_1} &= B + C\,\Delta Q_1 \\ \frac{\Delta s''}{\Delta Q_2} &= B + C\,(2\,Q_1 + \Delta Q_2) \\ \frac{\Delta s'''}{\Delta Q_3} &= B + C\,(2\,Q_2 + \Delta Q_3) \\ &\cdots\cdots\cdots \\ \frac{\Delta s^{(i)}}{\Delta Q_i} &= B + C\,(2\,Q_{i-1} + \Delta Q_i) \end{aligned}\right\} \quad \ldots\ldots (21)$$

Taking the differences between successive pairs of equations,

$$\left.\begin{aligned} \frac{\Delta s''}{\Delta Q_2} - \frac{\Delta s'}{\Delta Q_1} &= C\,(\Delta Q_1 + \Delta Q_2) \\ \frac{\Delta s'''}{\Delta Q_3} - \frac{\Delta s''}{\Delta Q_2} &= C\,(\Delta Q_2 + \Delta Q_3) \\ &\cdots\cdots\cdots \\ \frac{\Delta s^{(i)}}{\Delta Q_i} - \frac{\Delta s^{(i-1)}}{\Delta Q_{i-1}} &= C\,(\Delta Q_{i-1} + \Delta Q_i) \end{aligned}\right\} \quad \ldots\ldots (22)$$

Considering log t to be variable, Eqs. 21 are the equations of a series of parallel straight lines whose spacings are given by Eqs. 22. Just as the ratio of discharge to drawdown is termed "specific capacity," the ratio of drawdown

to discharge may be termed "specific drawdown." Similarly, the ratio of an increment of drawdown to the increment of discharge producing it may be termed "specific incremental drawdown." Eqs. 21, then, give the specific incremental drawdown as a function of time, the factor B increasing with time while the factor C remains constant.

The factor C may be determined from any one of Eqs. 22. Then, knowing C, there may be determined the difference:

$$\frac{\Delta s'}{\Delta Q_1} - \frac{\Delta s^{\circ}}{\Delta Q_0} = C\,(\Delta Q_0 + \Delta Q_1) \quad \ldots\ldots (23)$$

In Eq. 23, $\frac{\Delta s^{\circ}}{\Delta Q_0}$ is an abbreviation for the limiting value that the specific incremental drawdown approaches as the discharge increment approaches zero and as the well loss consequently becomes negligible. Subtracting Eq. 23 from the first of Eqs. 21, recalling Eq. 14, and keeping in mind that $\Delta Q_0 \rightarrow 0$:

$$\frac{\Delta s^{\circ}}{\Delta Q_0} = B = \frac{1}{4 \pi T}\left(\log_e \frac{t}{t^*_w} - 0.5772\right) \quad \ldots\ldots (24)$$

If the values of specific incremental drawdown given by Eqs. 21 are plotted against log t, as suggested previously a series of straight lines is obtained. Eq. 24 can be plotted on the same graph as a straight line parallel to the others and spaced with reference to the first of the other straight lines in accordance with Eq. 23. Inasmuch as the C-term is lacking in Eq. 24, that equation simply expresses the component of drawdown that occurs in the formation. From its intercept with the zero-drawdown line, then, the effective radius of the pumping well may be determined by the following modification of Eq. 17:

$$r^2_w = 2.25\,\frac{T\,t}{S} \quad \ldots\ldots (25)$$

In Eq. 25, S can be determined from observations in a near-by observation well at a known distance r, as shown in Fig. 4.

Data from Multiple-Step Drawdown Test

Figs. 7 and 8 give data from a multiple-step drawdown test run in August, 1943, at Bethpage, Long Island, N. Y. The well that was tested was gravel packed and had 50 ft of 8-in. screen with a No. 60 slot, and with bottom at 350 ft. It was equipped with a 1,200 gal per min deep-well turbine pump. The flow was metered with a propeller-type meter in the discharge line. Water levels inside the casing were measured by an air line with a pressure-gage reading in pounds per square inch. During the first period of the test, measurements of depth to water were made with a weighted steel tape. Thereafter it was not possible to lower the tape to the water surface.

This test had four periods of approximately one hour's duration each. By timing the dial on the nonrecording flow meter with a stop watch, it was possible to secure virtually instantaneous readings of the discharge. Actually, the pumping rate declined slightly during each period of the test as the constantspeed pump adjusted itself to the lowering water level in accordance with the head-discharge characteristic. As these variations were slight, only the average discharge for each period is shown in Fig. 7. Readings of the air-line pressure gage (converted) are indicated by ×'s. The circles plotted one hour after the beginning of each period are interpolated points that are carried over to the discharge-drawdown diagram (Fig. 7(*c*)). Curve A, drawn through these points, is the type of curve ordinarily obtained, drawdown readings being taken only at the end of each period of the test. Customarily, the specific capacity is determined from an average secant of curve A passing through the origin.

Values of drawdown increments taken from Fig. 7(*b*), divided by corresponding increments of discharge, are plotted in Fig. 8 against appropriate values of

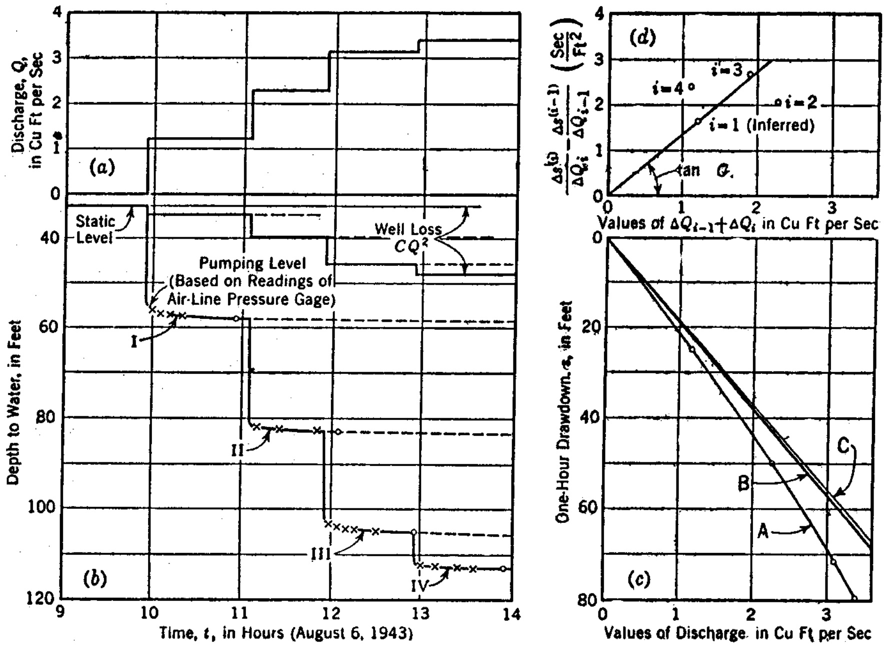
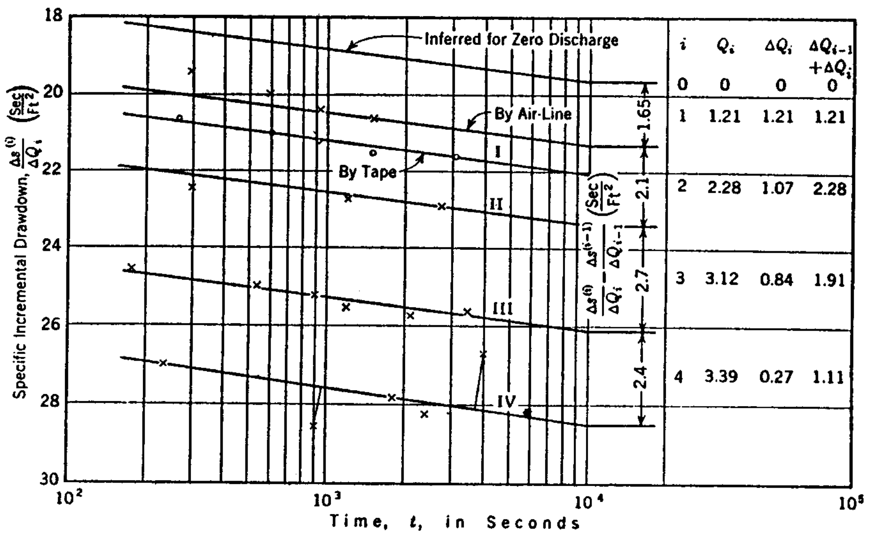


FIG. 7.—DETERMINATION OF WELL LOSS WHEN EFFECTIVE WELL RADIUS IS NOT KNOWN

time on a logarithmic scale. Using the tape readings during the first period as a guide, the slope of the several lines (which theoretically are parallel) is determined. The spacing of the lines in units of feet per cubic feet per second is given in the first column of the tabulation in Fig. 8. Also tabulated are the numbers of the periods, the discharge during each period the increment of discharge at the beginning of each period, and the sums of neighboring increments of discharge. In accordance with Eqs. 22. data in the first and last columns of this tabulation are plotted in Fig. 7(*d*) to determine C. The three experimental points, for i = 2, 3, and 4, show considerable scattering partly



i	Q_i	ΔQ_i	$\Delta Q_{i-1}+\Delta Q_i$
0	0	0	0
1	1.21	1.21	1.21
2	2.28	1.07	2.28
3	3.12	0.84	1.91
4	3.39	0.27	1.11

FIG. 8.—SEMILOGARITHMIC GRAPH OF DATA FROM FIG. 7(*b*)

perhaps because of the insufficiency of the theory but more likely because of inaccuracies of the air-line pressure-gage readings. From the slope of the straight line drawn through the center of mass of the three points, C is found to be 1.35 $\left(\frac{\sec^2}{ft^5}\right)$

Knowing C, the amount of well loss during each period of the test may now be determined. For the fourth period of the test, when the discharge was 3.39 cu ft per sec (1,520 gal per min), the well loss is computed as 15.5 ft. During the earlier periods of the test, when the discharge was smaller, the well loss was correspondingly smaller, as indicated in Fig. 7(*a*). Subtracting the appropriate value of well loss from each point plotted in Fig. 7(*c*), curve B is obtained. This curve then represents the discharge-drawdown relation for zero well loss. The straight line C (Fig. 7(*c*)) represents the relation between discharge and drawdown that would be obtained with zero well loss if separate one-hour tests were run at each rate, starting from rest with long intermediate periods of shutdown.

Theoretically, it should be possible to determine the transmissibility of the bed from the slope of the straight lines in Fig. 8; but the situation is complicated somewhat by the lenticular structure of the material penetrated by the well. Although the sand within a few hundred feet of the well is definitely confined, at greater distances it is effectively although somewhat circuitously interconnected with overlying beds of sand. If the setup were more nearly ideal and if there were a near-by observation well from which to determine the value of S, the effective radius of the pumping well might be determined.

Summary

Because of limitations of the data from the two examples given in the paper, a composite of both examples is needed to illustrate the theory of the multiple-step drawdown test completely. With this in mind, the practical application of the theory can be summarized somewhat as follows.

(1) The test should be run following a period in which the well has been inactive, beginning at a fraction of the capacity of the pump and increasing the discharge in steps, each of which is a fair fraction of the pump capacity (Fig. 7). During each period of the test the metered discharge should remain essentially constant. (Small variations in discharge arising from the automatic adjustment of the pump to the declining water level when pumping against a constant discharge pressure are permissible.)

(2) During each period of the drawdown test, frequent readings of the drawdown should be made by air line or, if possible, preferably by a steel tape or by an electric-contact device. If an air line is used, care should be taken to use a reliable pressure gage that has been calibrated and to read it to the nearest fifth or tenth of a scale division.

(3) Frequent drawdown readings should also be taken in one or more observation wells tapping the same sand. If the screen of the pumping well does not completely penetrate the sand, the nearest observation well for this purpose should not be closer than about twice the sand thickness from the pumping well.

(4) Plot the data obtained under item (3) on a semilogarithmic graph (Fig. 4), using increments of drawdown against logarithm of time Determine the transmissibility from the slope of the straight lines and the storage coefficient from their average intercept.

(5) Plot the data obtained under items (1) and (2) on rectangular coordinates (Figs. 7(*a*) and 7(*b*)). Extrapolate the drawdown curve for each period through the following period to determine the increments of drawdown.

(6) Plot values of specific incremental drawdown against the logarithm of time on semilogarithmic paper (Fig. 8). Draw parallel straight lines through the plotted points. (Extensions of these straight lines should check the extrapolations on the other graph. Secondary adjustments may be made to improve the extrapolations.)

(7) Plot differences of specific incremental drawdown given by neighboring lines against the sum of neighboring discharge increments (Fig. 7(*d*)). Determine C from the slope of the straight line through the origin and through the center of mass of the plotted data.

(8) Compute the well loss for each period of the test from $C\,Q^2$.

(9) Infer the limiting straight-line relation for zero discharge (Fig. 8) and from its intercept with the zero-drawdown line, using the value of storage coefficient determined under item (4), compute the effective radius of the well.

If the storage coefficient and transmissibility of the bed and the effective radius of the well are determined, it is possible to compute the resistance, B, at any time. Knowing the factor C, it is possible to compute the well loss. Combining the two, the total drawdown in the pumping well may be determined for any time and for any pumping rate.

In extensive artesian aquifers such predictions of drawdown are often trustworthy for periods of several months or a few years, but longer-term predictions must be based upon further consideration and closer evaluation of the outer "boundary conditions" of the aquifer. In local artesian beds this type of analysis may be required even for short-term predictions. In either case the concepts and procedures advanced in this paper should constitute useful implements, although not displacing in any way that knowledge of the geology and hydrology of an aquifer that is so necessary for a complete understanding of its performance.

The procedure itemized in this "Summary" should make possible, at any time during the life of a well, the accurate determination of both components of its specific drawdown, thus, for example, facilitating the recognition of the effects of encrustation of the screen or sand packing of the gravel wall, which too often have been ascribed to "depletion of the sand." Furthermore, it should enable the evaluation of the effectiveness of gravel packing and of the various development operations practiced in well construction. Through the accumulation of data, as wells are developed and placed in operation, this procedure should greatly aid in the selection of the proper gravel size and the appropriate screen opening so that the efficiency of wells will be increased and much needless waste of pumping energy will be prevented. Similarly, where the gravel wall is not called for, unnecessary expenditures for this type of construction may be avoided by referring to cases that have been tested under similar circumstances. Through predictions of the trend of pumping levels with time, proper selection of the pump and motor may be made that will give optimum performance throughout the life of the well, thus avoiding the wasteful practice of operating a pump with its discharge throttled to keep within the limits set by the diminishing capacity of the well.

The decline in production of oil wells is perhaps even more troublesome than that in the production of water wells. Often, it is difficult to determine whether the decline is due merely to depletion of the reservoir or whether it is due to the plugging of the perforations in the "liner," to the transportation of the fines, to the deposition of asphaltic substances, or to other causes. With some modifications the procedure outlined in this paper can be applied to oil wells as an aid in answering such questions. However, more accurate determinations of fluid level would be required than are now generally feasible while pumping steadily at different rates of production.

Acknowledgment

The writer wishes to express his appreciation of their helpfulness to O. E. Meinzer, chief of the Ground-Water Division, Water Resources Branch, United States Geological Survey and to M. L. Brashears, Jr., district geologist, in charge of ground-water investigations in New York and New England, under whose direction he worked during the development of this paper.

APPENDIX. NOTATION

The following letter symbols conform essentially with American Standard Letter Symbols for Hydraulics (ASA—Z10.2—1942) and with ASCE *Manual of Engineering Practice No. 22* on "Soil Mechanics Nomenclature"

B = "hydraulic resistance" of formation, head loss per unit discharge;
b = thickness of confined sand bed;
C = coefficient in term $C\,Q^2$ expressing "well loss," a component of drawdown, the other term of which is $B\,Q$;
k = transmission constant, or "coefficient of permeability";
n = porosity of sand;
Q = discharge of well;
ΔQ_i = increment of discharge; i = 1, 2, 3,
r = radial distance from axis of well;
r_w = "effective radius" of well;
S = "coefficient of storage" = $(b/V)(dV_w/ds)$;
s = drawdown at distance r, the difference between initial head and head at time t at that distance;
s_w = drawdown at r_w, according to theoretical logarithmic distribution;
$\Delta s^{(i)}$ = increment of drawdown produced by ΔQ
$\Delta s^{(i)}/\Delta Q_i$ = "specific incremental drawdown" during i^{th} period of test;
$\Delta s^\circ/\Delta Q_0$ = limiting value of specific incremental drawdown for discharge approaching zero (= B);
T = "transmissibility" of sand bed = $k\,b$;
t = time;
t^* = "inflectional time" = $r^2\,S/4\,T$;
u = $r^2\,S/4\,T\,t = t^*/t$, a nondimensional variable;
V = volume;
V_w = volume of water;
$W(u)$ = "well function" of u, or the negative "exponential integral" of $-u$, for which tables are available;
α = "compressibility" of solid skeleton of sand bed, relative decrease in thickness per unit increase of vertical component of compressive stress in sand bed;
β = compressibility of water in sand bed;
β' = apparent compressibility of water = $\beta + \alpha/n$; and
γ = specific weight of water.

DISCUSSION

N. S. Boulton,[11] Esq.—The importance of carefully recording both the small variations in pumping level, which may occur during pumping tests at constant discharge, and the duration of the test, are appropriately stressed in this paper. From such information it is possible to predict, as the author has shown, the probable steady decline in specific capacity "for periods of several months or a few years" when the well is pumped at constant discharge. It is important to remember, however, that the accuracy of this prediction depends essentially on the assumption that the compressibility of the aquifer (which enters into the coefficient of storage) has the same value for the very small pressure releases which occur at large distances from the pumped well as for the comparatively large pressure releases near to the well. It would be appreciated if the author could present evidence in support of this assumption, based on long-period observations of declining well levels. In addition, it would be interesting to know whether the author has been able to check the values for "well loss" by direct estimates of the pipe friction loss as the water flows inside the well casing and also of the loss of head due to the screen.

For the fourth period of the test at Bethpage, Long Island, N. Y., the depth of water in the well was apparently about 238 ft. Allowing for the water entering the well uniformly along the bottom 50 ft, a reasonable estimate (from a usual formula) for the head lost in pipe friction in the 8-in.-diameter tube is about 10.5 ft, including 1.5 ft for the velocity head. The computed well loss (see heading, "Data from Multiple-Step Drawdown Test") is stated to be 15.5 ft, which leaves 5 ft for the loss due to the screen. It is easy to calculate the latter loss on the assumption of flow through a uniform permeable medium outside the screen to which Darcy's law may be applied. Thus, for long vertical slots spaced equally around the circumference of the well, it can be shown from the potential solution for the flow net that the head loss due to the restricted inlet area provided by a slotted tube is closely given by:

$$h = \frac{Q}{2\,\pi\,N\,b\,k} \log_e \left(\frac{2}{1 - \cos \nu\,\pi}\right) \quad \text{(26)}$$

in which N is the number of vertical slots around the circumference of the tube; and ν is the slot-width ratio or width of slot divided by the distance between the centers of two adjacent slots.

According to Eq. 26, the head loss is proportional to the discharge and, for a given slot-width ratio, inversely proportional to the number of slots.

If Q = 3.39 cu ft per sec and b = 50 ft, as in the fourth period of the Bethpage test, and if k = 0.004 ft per sec (as deduced from Fig. 8), assuming $\nu = \frac{1}{4}$ (since the dimensions of the slotted tubing are not given in the paper), it is found on substitution in Eq. 26 that $h = 5.2/N$ ft. For one hundred slots, each 0.063 in. wide, uniformly spaced around the circumference, h = 0.052 ft which is negligible. On the other hand, if the slots are arranged in batteries

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numbering, say, ten in the circumference, the batteries being 0.5 in. wide with 2 in. between them, $h = 0.52$ ft which is still small.

It should be emphasized that this calculation makes no allowance for any clogging of the slots. Such clogging may account for the discrepancy between the small calculated screen loss and the value of 5 ft deduced from the test result.

Carl Rohwer,[12] M. ASCE.—The serious depletion of ground-water supplies in many areas during World War II has focused attention on the problems of ground-water hydrology. In this connection the investigations of the engineers of the Water Resources Branch of the U. S. Geological Survey are adding important information regarding the characteristics of wells and the capacity of ground-water formations. The analysis of drawdown tests of artesian wells by the author is a valuable contribution to this subject.

The writer is in agreement with the objectives of the author's investigations but he is of the opinion that the analysis of the problem would have been simplified if some of the factors that have only a slight effect on the results had been ignored. Under most conditions met with in the field of engineering the compressibility of water can be ignored. The coefficient is approximately 4×10^{-6} per pound pressure at ordinary temperatures and pressures. A reduction in pressure of 10 lb per sq in. would increase the volume of 1 cu ft of water by only 0.00004 cu ft, a difference of 1 in 25,000. In view of the large unavoidable errors involved in other measurements it seems that this factor could well be neglected. The same may be stated of the compression of the aquifer. As indicated by the author (see heading, "Application of Theory to a Simple Drawdown Test"), the combined effect of compressibility of the water-bearing formation is only five times the actual compressibility of water. Consequently, the combination of these two factors would produce a change of only 1 in 5,000 for a drop in pressure of 10 lb (approximately 23 ft). If the change in pressure were increased to 100 ft the effect produced by the compressibility of the water and aquifer would not be significant.

In reference to the tests on a shallow well at Meadville, Pa., the author states in the sentences following Eq. 18, that:

> "The foregoing calculations indicate that $B\,Q$ in this case was about 19.5 ft, after 24 hours of continuous pumping. The observed drawdown in the pumping well at that time was 48.0 ft, leaving 28.5 ft for the well loss."

Such a large well loss seems unusual for an inflow of 1,350 gal per min through 15 ft of 18-in. screen unless the screen were badly encrusted or improperly perforated. Immediate steps should be taken to improve the performance of the screen in this well.

In the solution of problems involving many variables of which only a few can be determined by direct measurement, the use of multiple equations provides a method of determining the unknowns. However, there are difficulties inherent in this method which may lead to contradictory or inconsistent results.

[12] Senior Irrig. Engr., Div. of Irrig., Soil Conservation Service, U. S. Dept. of Agriculture, Fort Collins, Colo.

As shown by the author in reference to the determination of C (see heading, "Data from Multiple-Step Drawdown Test"), there is considerable scattering in the values obtained for the "specific incremental drawdown," Fig. 7(d). No doubt, this is due in part to the inaccuracies in the drawdown readings. If this method were used on problems in which all readings could be made accurately, consistent results should be expected. Since this is not generally true, the multiple-equation method results in solutions in which the final answers may have errors greatly in excess of the observed data. The writer is not aware of the mathematical basis for this assumption, but he observed the same effect when attempting to use a similar method to determine the values of the factors involved in the seepage from canals. The conclusion was reached that, in the elimination of variables from the series of equations by subtraction, the variables eliminated were forced to conform exactly to the law; and, as a result, all the discrepancies accumulated and finally appeared in the solution of the unknown. A solution based on another pair of equations may, therefore, yield a result widely different from the first one.

Since the author has had the opportunity to observe how the solutions vary when he uses different equations it would be of interest to study the mathematical principles causing the variations. No doubt rules could be formulated which would make it possible to obtain more consistent results from the observed data. Such an analysis would be useful in the solution of problems in other fields of engineering.

R. M. Leggette,[13] Affiliate, ASCE—Although it covers a highly technical subject, this paper clearly demonstrates the practical importance of a number of factors of well design. It seems desirable to emphasize these practical considerations because they are often given too little attention. Frequently water works men and well-drilling contractors greatly belittle or fail to recognize the magnitude of what Mr. Jacob calls "well loss."

It is obvious, of course, that the water level in a pumping well must be lower than the water level immediately outside the well. In many wells, much of this difference in head is screen friction loss which results from the use of a poorly designed screen. This difference in head is sometimes presumed to be only a few inches, or a fraction of a foot; however, actual observations have shown that in some wells the well loss is a considerable part of the total drawdown. Thus, from the point of view of economy of operation, well loss may be an important factor.

The paper also indicates the desirability of increasing the effective radius of a "sand and gravel" well by development to remove the fine material surrounding the screen, or by artificial gravel packing. It should be noted that the advantages of development or gravel packing may be largely overcome if an inefficient well screen is used.

The process of development by surging, swabbing, and brushing is being used more and more in uncased wells (rock wells), the walls of which apparently become "mudded up" during the drilling process. This clogging of the uncased wall of the well has the same effect as an inefficient well screen in a "sand

[13] Cons. Ground-Water Geologist, New York, N. Y.

and gravel" well. The well loss in many wells of this type has been greatly reduced by development.

From the point of view of economy of pumping well water, this paper indicates the following: A well should be of large diameter; it should be extensively developed; and, if a well screen is used, it should be designed so as to produce a minimum of screen friction loss.

M. R. Lewis,[14] M. ASCE.—A valuable method has been presented, in this paper, for analyzing the capacity of artesian wells in spite of the necessary assumptions that there is uniformity in the aquifer and that the total supply to the well is drawn from the aquifer by the release of elastic forces. Such theoretical or mathematical treatments of the flow of fluids assist greatly in the understanding of practical problems even though the latter seldom are based on ideal conditions.

The multiple-step test proposed by the author should give very useful information on the points mentioned in the "Summary" wherever the assumptions are approximately fulfilled. It is hoped that the author will explore the possibility of a similar analysis under other conditions. Two important types of wells that might be studied are those of a simple water-table situation and those in which the bed overlying an aquifer is relatively impermeable but permits recharge from the soil surface surrounding the well.

It appears to the writer that two other factors besides the compaction of the aquifer are important elements in making the "apparent compressibility, β'," greater than the compressibility of water, β. These are (a) the increase in volume of the solid material of the aquifer because of the reduced hydrostatic pressure and (b) the reduction in the pore space because of the deformation of the solid particles by reason of the increased pressure on the mineral skeleton. In his earlier paper,[8] the author mentioned these factors but, apparently, considered them to be of negligible importance. Whether they are, or are not, important makes no difference in the author's analysis.

C. E. Jacob,[15] Assoc. M. ASCE.—Although few in number and brief, the discussions have added much to the paper and, moreover, have suggested the direction that further work might profitably take. The writer wishes to thank those who have contributed.

The question raised by Mr. Boulton is a pertinent one—regarding the need for evidence to support the assumption that the compressibility of the aquifer has the same value near to, and far from, the pumped well despite the wide range of pressure release. The writer knows of no close observations of long-period decline under constant and continuous pumping that might clarify this problem. Of course, it is to be expected that an unconsolidated sand would have a variable compressibility, depending on the rate and magnitude of the loading occasioned by the release of pressure. Moreover sight should not be lost of the fact that the flexural rigidity of the overlying beds complicates the problem, especially in the immediate vicinity of pumped wells tapping deep aquifers. Fortunately, however, as long as the compressibility (thus modified)

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may be assumed reasonably constant in time at a given distance the uncertainties arising from ignorance of its absolute value are reflected only in the degree of approximation of r_w. Actually, in predictions of future drawdown based on the theory of an elastic aquifer, the product $S\,r^2_w$ is used. This product is determined empirically and it is not necessary to break it down into S and r_w, except to compare different wells. If S varies with t, for one reason or another, then some other theory must be used or the elastic theory must be modified.

Mr. Boulton's estimate of 10.5 ft for the friction and velocity head losses accompanying the upward flow inside the well casing of the Bethpage well narrows down the well loss to that part that may be termed "screen loss." His calculation of the "convergence loss" under laminar flow into an assumed system of vertical slots shows that such loss would be quite small. Actually the openings in the screen are in the form of a helix, but the "convergence loss" there should be of the same or even of a smaller order of magnitude. Clogging of the slots is a distinct possibility, as pointed out by Mr. Boulton. Furthermore, departure from laminar flow may begin as the water passes through the screen openings, especially if they are clogged.

By calculation Mr. Rohwer shows that the combined relative compression of the aquifer near Meadville, Pa., and its contained water is only about 1 in 5,000 for a 23-ft drop in head. He states:

> "In view of the large unavoidable errors involved in other measurements it seems that this factor [compressibility of water] could well be neglected. The same may be stated of the compression of the aquifer."

Perhaps Mr. Rohwer has in mind estimating the ultimate or steady-state discharge of wells in a limited aquifer, in which case the volume of water derived from artesian storage might soon become small by comparison with the volume drawn through the aquifer from an outer boundary. However, until that steady state of flow is established, water is withdrawn from storage through its elastic expansion and through the concomitant compression or compaction of the aquifer. Indeed, in extensive and deep-lying aquifers such as the Dakota sandstone, the supply may be furnished entirely from storage for decades. Because of the tremendous volume of water in such an aquifer and because of the sizable lowering of head that may be produced, the total volume of water withdrawn from storage through wells may itself reach an astounding magnitude. In any event, however insignificant this factor may appear, the study of the transient behavior of an elastic aquifer, on which the paper is based, requires an appraisal of its magnitude.

With reference to the large well loss of 28.5 ft at 1,350 gal per min in the well near Meadville, Pa., the suggestion is offered by Mr. Rohwer that the screen may be encrusted or improperly perforated. As the well was newly constructed when tested, it would seem that sand packing of the gravel envelope and of the screen might be the explanation. The formation is a uniform fine sand—difficult to handle in drilling and developing a well.

Mr. Rohwer adds a valuable point in remarking on the unavoidable magnification of errors through elimination of variables by subtraction from the series of equations. He justifiably emphasizes the need for care in treating such data as those presented in the paper. Whereas from a theoretical standpoint, with ideal data, the procedure outlined in the paper is sound; in practice it needs modification. Higher precision of measurement may warrant the assumption that the drawdown obeys the law $s_w = B\,Q + C\,Q^n$, introducing a third unknown, the exponent n (<2), to be determined by trial-and-error computation, or by graphical procedure together with the coefficients B and C.

An important point is raised by Mr. Leggette—that the advantages of gravel packing or developing a well may be offset by poor design or improper choice of screen. The writer feels that amassing empirical values of C and of r_w, together with pertinent data on the details of design and construction of wells, may eventually make possible the accurate appraisal of these various factors. The selection of screen type, slot opening, and gravel size—and even the determination of whether or not a gravel envelope is required—may be lifted from the realm of guesswork to a rational plane through the future study of existing and newly constructed wells and through the measurement of their characteristics of performance, due consideration being given to the transient behavior of the aquifer.

In emphasizing the magnitude of well losses, condemnation of the well driller is not intended, for much of the friction loss in and near a well is unavoidable and will never be entirely eliminated. Nevertheless, it behooves the engineer and the well-drilling contractor alike to strive for as efficient design and construction as possible to meet the stringencies of economic demands. The points summarized by Mr. Leggette thus are objectives toward which progress should be made.

Mr. Lewis suggests that similar analyses be made for unconfined flow under simple water-table conditions, and for confined flow in which recharge from the soil surface occurs through a relatively impermeable confining bed. To the writer's knowledge a satisfactory analysis of nonsteady unconfined flow has not been given. Even in the case where the storage coefficient (S, ultimately approaching "specific yield") is constant, there are insuperable difficulties. Only by analogy to confined flow, and then in cases where the maximum drawdown is but a small fraction of the initial depth of flow, has an approximate solution been obtained. It may be stated, however, that even in the absence of well losses the specific capacity of a water-table well would vary with the discharge, the curve of drawdown versus discharge at constant time being a parabola under certain approximative assumptions. Further work should be done on this problem, both in the laboratory and in the field.

A solution has been given for the nonsteady radial flow toward a steadily discharging well in a leaky confined aquifer.[16] The leakage is assumed proportional to the drawdown. Whether this is exactly the condition Mr. Lewis has in mind is not known, but in the early phase of a transient state such a system acts like an ideal elastic aquifer without leakage Accordingly, a short multiple-step drawdown test could be analyzed under those conditions on the basis of the elastic theory, although long-term predictions would consider the leakage.

[16] "Radial Flow in a Leaky Artesian Aquifer," by C. E. Jacob, *Transactions*, Am. Geophysical Union, Vol. 27, 1946, Pt. II, pp. 198–205.

CALCULATION OF C

JACOB (1946)

1. STEP 1 : $t_0 < t < t_1$, $Q = Q_1$, $\Delta Q_1 = Q_1$

$$s_w = \Delta s_1 = \frac{\Delta Q_1}{4\pi T}\left(\ln\left\{ \frac{4T(t-t_0)}{r^2 S} \right\} - 0.5772 \right) + C Q_1^2 \qquad \text{—(19A)}$$

STEP 2 : $t_1 < t < t_2$, $Q = Q_2$, $\Delta Q_2 = Q_2 - Q_1$

$$s_w = \frac{\Delta Q_1}{4\pi T}\left(\ln\left\{ \frac{4T(t_1-t_0)}{r^2 S} \right\} - 0.5772 \right)$$

$$+ \frac{\Delta Q_2}{4\pi T}\left(\ln\left\{ \frac{4T(t-t_1)}{r^2 S} \right\} - 0.5772 \right) + C Q_2^2$$

and $\Delta s_2 = s_w - \Delta s_1(t_1)$

$$= \frac{\Delta Q_2}{4\pi T}\left(\ln\left\{ \frac{4T(t-t_1)}{r^2 S} \right\} - 0.5772 \right) + C\left(Q_2^2 - Q_1^2\right) \qquad \text{—(19B)}$$

STEP 3 : $t_2 < t < t_3$, $Q = Q_3$, $\Delta Q_3 = Q_3 - Q_2$

$$s_w = \frac{\Delta Q_1}{4\pi T}\left(\ln\left\{ \frac{4T(t_1-t_0)}{r^2 S} \right\} - 0.5772 \right)$$

$$+ \frac{\Delta Q_2}{4\pi T}\left(\ln\left\{ \frac{4T(t_2-t_1)}{r^2 S} \right\} - 0.5772 \right)$$

$$+ \frac{\Delta Q_3}{4\pi T}\left(\ln\left\{ \frac{4T(t-t_2)}{r^2 S} \right\} - 0.5772 \right) + C Q_3^2$$

and $\Delta s_3 = s_w - s_w\,(t_2)$

$$= \frac{\Delta Q_3}{\cdot} \left(\ln\left\{ \frac{4T(t-t_2)}{\cdot r^2 S} \right\} - 0.5772 \right) + C\left(Q_3^2 - Q_2^2\right) \qquad \text{—(19C)}$$

The generalization is :

$$\Delta s_{(i)} = s_w - s_w\left(t_{(i-1)}\right)$$

$$= \frac{\Delta Q_{(i)}}{4\pi T} \left(\ln\left\{ \frac{4T\left(t - t_{(i-1)}\right)}{r^2 S} \right\} - 0.5772 \right) + C\left(Q_{(i)}^2 - Q_{(i-1)}^2\right)$$

—(20)

Note : Jacob refers to $\left(t - t_{(i-1)}\right)$ as $t^{(i)}$

2. Divide through by the respective increments of discharge :

STEP 1 : $$\frac{\Delta s_1}{\Delta Q_1} = \frac{1}{4\pi T} \left(\ln\left\{ \frac{4T(t-t_0)}{r^2 S} \right\} - 0.5772 \right) + C\,Q_1 \qquad \text{—(21A)}$$

STEP 2 : $$\frac{\Delta s_2}{\Delta Q_2} = \frac{1}{4\pi T} \left(\ln\left\{ \frac{4T(t-t_1)}{r^2 S} \right\} - 0.5772 \right) + C\,\frac{\left(Q_2^2 - Q_1^2\right)}{\Delta Q_2}$$

Now $Q_2 = Q_1 + \Delta Q_2$

$$\therefore \quad \frac{\Delta s_2}{\Delta Q_2} = \frac{1}{4\pi T} \left(\ln\left\{ \frac{4T(t-t_1)}{r^2 S} \right\} - 0.5772 \right) + C\left(2\,Q_1 + \Delta Q_2\right)$$

—(21B)

STEP 3 : $$\frac{\Delta s_3}{\Delta Q_3} = \frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t-t_2)}{r^2 S}\right\} - 0.5772\right) + C\,\frac{(Q_3^2 - Q_2^2)}{\Delta Q_3}$$

Now $Q_3 = Q_2 + \Delta Q_3$

$$\therefore \quad \frac{\Delta s_3}{\Delta Q_3} = \frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t-t_2)}{r^2 S}\right\} - 0.5772\right) + C\left(2Q_2 + \Delta Q_3\right) \qquad \text{—(21C)}$$

The generalization is :

$$\frac{\Delta s_{(i)}}{\Delta Q_{(i)}} = \underbrace{\frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t-t_{(i-1)})}{r^2 S}\right\} - 0.5772\right)}_{\text{Jacob calls this } B} + C\left(2Q_{(i-1)} + \Delta Q_{(i)}\right) \qquad \text{—(2D)}$$

3. Take the difference between each successive pair, at the ends of the steps

STEP 2 − STEP 1 :

$$\frac{\Delta s_2}{\Delta Q_2} - \frac{\Delta s_1}{\Delta Q_1} = \frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t_2-t_1)}{r^2 S}\right\} - 0.5772\right) + C(2Q_1 + \Delta Q_2)$$

$$- \frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t_1-t_0)}{r^2 S}\right\} - 0.5772\right) - CQ_1$$

$$= \frac{1}{4\pi T}\left(\ln\left\{\frac{4T(t_2-t_1)}{r^2 S}\right\} - \ln\left\{\frac{4T(t_1-t_0)}{r^2 S}\right\}\right)$$

$$+ C\left(Q_1 + \Delta Q_2\right) \qquad \text{—(22A)}$$

GRAPHICAL AND THEORETICAL ANALYSIS OF STEP-DRAWDOWN TEST OF ARTESIAN WELL[1]

M. I. Rorabaugh,[2] Assoc. M. ASCE

ABSTRACT

The drawdown in an artesian well resulting from the withdrawal of water is made up of head loss resulting from laminar flow in the formation, and head loss resulting from turbulent flow in the zone outside the well, through the well screen, and in the well casing.

A graphical method for the empirical determination of laminar and turbulent head losses is given in this paper. The empirical method is compared and related to theoretical equations. A method is given for computing the head-loss distribution outside the pumped well for various pumping rates. Analysis is made of the variation of specific capacity with discharge and of the importance of well radius in well design.

INTRODUCTION

C. E. Jacob, in a paper entitled, "Drawdown test to determine effective radius of artesian well,"[3] presented a method for evaluating the head loss resulting from turbulent flow in the immediate vicinity of an artesian well by means of a step-drawdown test. The assumption was made that the flow near the well face would be turbulent, the head loss in the zone of turbulence being "approximately proportional to some higher power of the velocity approaching the square of the velocity"; whereas the flow in the remainder of the aquifer would be laminar, the aquifer loss varying directly as the discharge.

N. J. Lusczynski, in a study of which the results have not been published, developed a method using an observation well to eliminate correction for time effects. In that paper, the turbulent-flow exponent was considered as an unknown constant rather than approximated as a square relationship.

Graphical Method

Jacob used the equation

$$s_w = BQ + CQ^2 \tag{1}$$

in which s_w is the drawdown in the pumped well; B, the aquifer constant; C, the "well-loss" constant; and Q, the discharge.[4] As used in this paper, s_w is the drawdown in the pumped well for a constant discharge at a fixed

1. Published by permission of the Director, U. S. Geological Survey.
2. Dist. Engr., U. S. Geol. Survey, Ground Water Branch, Louisville, Ky.
3. Jacob, C. E., Drawdown test to determine effective radius of artesian well: Am. Soc. Civil Eng. Trans., vol. 112, paper no. 2321, p. 1047, 1947.
4. The letter symbols in this paper are defined where they first appear and are assembled alphabetically, for convenience of reference, in the Appendix.

time, pro d the well was in a static condition prior to each step. If the well is not allowed to recover between steps (changes in pumping rates), drawdown curves must be extended as was done by Jacob. In this paper s_w for any step is equal to the sum of the incremental drawdowns used by Jacob, $s_w = \Sigma(\Delta s_1 + \Delta s_2 + \text{-----}\ n)$. Jacob's equation can be put in the form

$$\frac{s_w}{Q} - B = CQ$$

or

$$\log\left(\frac{s_w}{Q} - B\right) = \log C + \log Q \qquad (2)$$

The latter is in the form of a straight-line equation and will plot as a straight line on log-log paper when Q is plotted against $(\frac{s_w}{Q} - B)$. At $Q = 1$, $C = \frac{s_w}{Q} - B$. The slope of the line is unity. On figure 1, line A is plotted from Jacob's solution ($B = 19$ sec./ft.^{2}, $C = 1.35$ sec.^{2}/ft.^{5}) by drawing a straight line at a 45° angle (slope = 1) through the point $Q = 1$, $\frac{s_w}{Q} - B = C = 1.35$. When the step-test data used by Jacob are plotted on this diagram, it is found that the data for the various steps plot a curve (parameter B = 19 on fig. 1) rather than a straight line. If the exponent for turbulent flow is allowed to stand as the square, points 2, 3, and 4 could be considered to be within the range of error of measurement, although these points indicate possible curvature. Point 1, determined at the lowest rate of discharge, might be discounted if it be assumed that turbulence was not fully developed at that rate.

If the exponent for turbulent flow is expressed as an unknown constant "n," a similar analysis may be used. The equation is then

$$s_w = BQ + CQ^n \qquad (3)$$

which rearranges to

$$\left(\frac{s_w}{Q} - B\right) = CQ^{n-1}$$

or

$$\log\left(\frac{s_w}{Q} - B\right) = \log C + (n - 1)\log Q \qquad (4)$$

An empirical solution is obtained by a graphical solution of the straight-line equation on log-log paper.

Assume values of B; for each value of B, plot $(\frac{s_w}{Q} - B)$ vs. Q on log paper. The solution is obtained when a value of B is found that produces a straight line. C is obtained from the intercept; i.e., where $Q = 1$, $C = \frac{s_w}{Q} - B$. The slope of the line equals $(n - 1)$, which produces a solution for n. Figure 1, based on Jacob's data, illustrates this method. Curves are shown for various trials of B. The final solution is $B = 20.4$ sec./ft.^{2}; C, from the Q intercept, $= 0.44$; and n, from the slope, equals 2.64. Units of CQ^n must be feet, so that the units of C are sec.^{n}/ft.$^{3n-1}$.

Table 1 shows original data and drawdown computed by both methods. Also shown are drawdowns for higher rates of pumping as might be computed for design of a pump.

TABLE 1

Comparison of observed drawdown and drawdown computed by two methods

Step	Discharge Q (cfs)	Observed drawdown s_w (feet)	$\frac{s_w}{Q}$ (sec./ft.^{2})	For B = 20.4 $(\frac{s_w}{Q} - B)$ (sec./ft.^{2})	Jacob BQ (ft.)	Jacob CQ^2 (ft.)	Jacob s_w (ft.)	Log-log BQ (ft.)	Log-log CQ^n (ft.)	Log-log s_w (ft.)
1	1.21	25.4	21.0	0.6	23.0	2.0	25.0	24.7	0.7	25.4
2	2.28	50.4	22.1	1.7	43.3	7.0	50.3	46.5	3.9	50.4
3	3.12	72.3	23.2	2.8	59.2	13.2	72.4	63.6	8.9	72.5
4	3.39	80.3	23.7	3.3	64.4	15.5	79.9	69.1	11.0	80.1
	4				76.0	21.6	97.6	81.6	17.1	98.7
	5				95.0	33.8	128.8	102.0	30.8	132.8
	6				114.0	48.6	162.6	122.4	50.2	172.6

Inspection of figure 1 and table 1 shows the following:

1. Drawdowns computed by either method are very close to the original data. The problem is one of writing the equation of a curve fitting four points. The data cover such a short range that any number of different curves could be applied. If the test covers the range of use, either solution may be applied. However, if computations must be made for discharge greatly in excess of those used in the test (for pump design or for determination of maximum yield), interpretation might be poor. Until better methods are devised, a margin of safety can be provided by computation by both methods, using the least favorable solution for the purpose.

2. Comparison of the formational portion of the drawdown (BQ) computed by the two methods shows that the log-log graphical method assigns a larger portion of the total drawdown to formational loss, and shows smaller "well loss" in the range of the data but greater "well loss" in the extension to higher discharges.

3. The graphical method emphasizes the need for accurate data. Small erros in discharge or drawdown measurements will cause relatively large errors in n, B, and C. This sensitivity is present in Jacob's and Lusczynski's methods but is not nearly so apparent as in the log-log graphical method.

4. The graphical method has a definite advantage over the mathematical equation. In applying the equation no allowance can be made for experimental error because the data must be treated in groups. Averaging results obtained by computing successive groups of data may yield poor results. By the graphical method a line is fitted to all the data. If one set of data is poor, it will fall off the curves and can be given less weight. In the mathematical method one poor reading could affect three successive sets of computations. It is generally recognized that graphical procedures are best for treating problems based on measurements of physical quantities in which errors are inherent. Rounding or adjusting the original data graphically before applying the equation may eliminate poor data; however, such a procedure must be done cautiously as adjusted data may lead to misinterpretation.

5. The graphical method is much less complicated than the mathematical. It can be applied treating the exponent as an unknown constant; or, if conditions warrant acceptance of the exponent as a square, the best fit of a 45° line will give a solution.

Magnitude of exponent "n"

Jacob's adoption of an exponent of 2 for turbulent flow is reasonable in view of the results of widely different hydraulic studies, and it may eventually prove to be an assumption that will produce usable results for many problems.

R. W. Stallman, Lusczynski, Jacob, W. F. Guyton, and the author have discussed this problem for several years. The graphical method has been applied to several tests and in each case the exponent has been greater than 2. Apparently, the explanation lies in the fact that past analysis has been based on the assumption that the critical radius is constant as discharge varies. It seems logical to assume that, at low rates of flow, no turbulence is present in the aquifer. As discharge is increased turbulent flow occurs first at the well face, and as discharge is further increased the boundary between laminar flow and turbulent flow moves outward from the well. The assumption that the head loss can be separated into BQ, representing laminar-flow losses, and CQ^n, representing turbulent-flow losses, for all discharge rates is therefore erroneous; however, the empirical solution is usable for certain problems, as will be demonstrated later in this paper.

Figure 2 is a sketch showing drawdowns near a pumped well for several rates of discharge if the area of turbulence be allowed to vary with discharge. Shown also is the $BQ + CQ^n$ empirical distribution of head. The graphical method is based on the assumption of a constant radius, r_B, for the boundary between "well loss" and formation loss. Only at one discharge rate, Q_B, would the radius r_B coincide with the true boundary between laminar and turbulent flow. At a higher discharge rate the drawdown attributed to formational loss, BQ, carries the logarithmic distribution of head into the turbulent zone as shown by the dashed line in the sketch. The term for "well loss," CQ^n, represents the loss from the intersection of the logarithmic distribution curve with the radius r_B to the pumping level in the well. It includes the loss caused by turbulent flow between r_B and the well face (which varies approximately as the square of the discharge), plus the excess of actual turbulent-flow loss over assumed laminar-flow loss in the zone between r_B and the turbulent-laminar-flow boundary, plus the loss due to upward flow in the well, s_f (which also varies approximately as the square of the discharge). It can be seen, then, that the exponent in the term CQ^n is not a true expression of the variation of head loss with discharge for conditions of turbulent flow. The exponent may be unity at very low rates of discharge, or it may be in excess of 2 for the case where the turbulent zone has a radius in excess of r_B. Jacob's assumption of the exponent of 2 would be true only for the discharge rate of Q_B; however, his assumption would yield a usable solution in cases where the turbulent flow up the well was large relative to the turbulent loss in the aquifer.

If laminar flow exists at low rates, the data should plot horizontally (slope of $n - 1 = 0$) on the log plot. Figure 3 shows data for a step test run southwest of Louisville, Ky., in 1945. The first two points plot as a horizontal line (slope = 0) indicating laminar flow. Since only two points are available for higher rates of discharge, it is not possible to evaluate the test. If the square relationship be assumed, the data may be interpreted as:

$$\text{For } Q < Q_c,\ s_w = BQ + C'Q \text{ (for laminar flow)}$$

$$\text{For } Q > Q_c,\ s_w = BQ + CQ^2 \text{ (for turbulent flow)}$$

where C is the "well-loss" constant for turbulent flow and C' the "well-loss" constant for laminar flow, and Q_c is the critical discharge below which laminar flow prevails. The critical discharge, Q_c, is determined approximately as 1.5 cfs from the intersection of the two straight lines. B is determined as 14.25; C = 1.54 from the intercept of the 45° line; and C' = 2.30 from the intercept of the horizontal line.

Reynolds numbers are computed[5] at the well face for the four steps as 9, 13, 17, and 23. These figures are necessarily approximate, but they do indicate that the assumption of laminar flow for the lower steps is not unreasonable. Tolman[6] indicates that laminar flow exists for $R < 10$.

5. Grain size determined initially by sieve analysis and modified by development of the well during construction to remove the finer-grained 60 percent of the particles. Reynolds number then computed as average grain size of remaining 40 percent of material times velocity times density divided by absolute viscosity. Velocity is used as discharge divided by cross-sectional area at screen.
6. Tolman, C. F., Ground water, p. 199, New York and London, McGraw Hill Book Co., Inc., 1937.

Figur 4 shows a solution of a test run northeast of Louisville in 1946. It is not hat the exponent is greater than 2, and that, if the square is used, the data plot a curve relative to the 45° straight-line solution. Graphical solution of a step test made by Lusczynski at Port Washington, N. Y., in 1947,[7] yields the solution, $s_w = 51.5\ Q + 10.5\ Q^{2.56}$. It may be significant that values of n are consistently in the range near 2.5. (Test by Jacob at Bethpage, Long Island, N. Y., in 1943, 2.64; Louisville test, 2.54; Port Washington test, 2.56; and tests at Houston, Tex., in 1949, 2.43 and 2.82.)

Development of Equation for Separating Laminar and Turbulent Head Losses

The following discussion is given in an attempt to clarify the use of the empirical equation ($s_w = BQ + CQ^n$), to shed some light on the limits of its use and to tie the empirical equation to theory.

The use of "effective radius" is a convenient but somewhat misleading method of handling well-loss problems. Many unknowns or indeterminate factors are thrown into this term. Effects of partial penetration, effects of changes in transmissibility near the well because of development, and many other factors only partly accounted for are included in the term. Since the physical well radius is usually known, this discussion will be based on the inside radius of the well or screen, r_n.

Referring again to figure 2, the drawdown in the well is expressed as

$$s_w = M - N + s_t + s_f \tag{5}$$

in which M represents the head loss at the well face according to laminar-flow theory, N is the head loss according to laminar-flow theory between the critical boundary and the well face, s_t is the head loss according to turbulent-flow theory between the critical boundary and the well face, and s_f is the head loss due to pipe friction in the well.

For artesian conditions, for a fully penetrating well, and after pumping at a constant rate for a time long enough to establish a steady-state condition, the difference in drawdown between two points in the aquifer r_1 and r_2 distant from the pumped well may be expressed

$$s_1 - s_2 = \frac{2.3\ Q \log \frac{r_2}{r_1}}{2\pi T} \tag{6}$$

where Q is the pumping rate, T the transmissibility, and s_1 and s_2 the drawdowns at points r_1 and r_2.

The term M at a fixed time, under equilibrium conditions, (assume homogeneous material and neglect effects of development) is expressed as

$$M = \frac{2.3\ Q \log \frac{r_e}{r_n}}{2\pi T} \tag{7}$$

where r_e is the intercept for zero drawdown on the semilog profile plot and r_n the inside radius of the screen.

7. Data in open file of the U. S. Geological Survey at Washington, D. C., and Mineola, N. Y.

The term N is expressed

$$N = \frac{2.3\ Q \log \frac{r_t}{r_n}}{2\pi T} \tag{8}$$

where r_t is the distance from the well center to the boundary between laminar and turbulent flow.

The distance to the boundary of the zone of turbulent flow (under the conditions set forth) should vary directly with discharge.

$$\frac{r_{t_1}}{Q_1} = \frac{r_{t_2}}{Q_2} \tag{9}$$

At low rates of flow no turbulent flow will exist in the aquifer or screen. The critical discharge at which turbulence is introduced at the well face is designated Q_c. The expression then is written

$$\frac{r_n}{r_t} = \frac{Q_c}{Q} \tag{10}$$

which applies only when $Q > Q_c$.

The expression for N may be modified by substitution to

$$N = \frac{2.3\ Q \log \frac{Q}{Q_c}}{2\pi T} \tag{11}$$

The term s_t, representing the head loss in the turbulent-flow zone between the laminar-turbulent boundary (r_t) and the inside face of the well (r_n), is approximated as follows:

Assume that the loss in head varies as the square of the velocity. The head loss across an increment dr is then expressed $ds = Ev^2 dr$, where E is the aquifer constant for turbulent flow. The velocity at any point is equal to discharge divided by area $v = Q/a$, and area equals $2\pi r m\theta$, in which m represents the height of the section through which flow is occurring, and θ is the porosity of the bed.

Substituting

$$ds = \frac{EQ^2 dr}{(2\pi r m\theta)^2}$$

Integrating between the limits r_t and r_n, we obtain

$$s_t = \frac{EQ^2}{(2\pi m\theta)^2}\left(\frac{1}{r_n} - \frac{1}{r_t}\right) \tag{12}$$

Substituting $r_t = \frac{Q\ r_n}{Q_c}$

$$s_t = \frac{EQ^2}{(2\pi m\theta)^2}\left(\frac{1}{r_n} - \frac{Q_c}{Q\ r_n}\right)$$

Simplifying

$$s_t = \frac{EQ}{(2\pi m\theta)^2\ r_n}(Q - Q_c) \tag{13}$$

For a given well the term $E/(2\pi m\theta)^2\ r_n$ may be replaced by a constant D

$$s_t = DQ\ (Q - Q_c) \tag{14}$$

The ter s_f, representing the friction loss caused by flow up the pipe, may be as ned to vary approximately as the square of the discharge

$$s_f = FQ^2 \tag{15}$$

in which F is a constant for friction loss due to turbulent flow up the well.

Bringing the various terms of head loss (equations 7, 8, 14, and 15) together, the equation for the drawdown in the pumped well is

$$s_w = \frac{2.3\,Q}{2\,\pi\,T} \log \frac{r_e}{r_n} - \frac{2.3\,Q}{2\,\pi\,T} \log \frac{Q}{Q_c} + DQ\,(Q - Q_c) + FQ^2 \tag{16}$$

At least one observation well is required in order to apply this equation. The term r_e can be computed if S and T are determined from a time-drawdown plot at one observation well, or can be obtained from the zero-drawdown intercept of a semilog profile plot if several observation wells are available. Q and s_w are measured during the step test and r_n is known.

Unknowns are D, F, and Q_c. If the step test is run at both high and low rates of discharge and if sufficient points are available, Q_c may be determined from the log-log plot as discussed in the first part of this paper and illustrated by figure 3. Q_c is obtained from the intersection of the horizontal straight line for laminar flow and the angular straight line for turbulent flow. If data cannot be obtained at low rates, Q_c cannot be obtained by that method.

Inspection of equation (16) shows that it is impossible to solve for D and F. However, the equation is useful for certain specific cases.

If the equation is written for each of two pumping rates Q_1 and Q_2 and the first equation subtracted from the second, the following equation is obtained:

$$\frac{s_2}{Q_2} - \frac{s_1}{Q_1} = \frac{2.3}{2\,\pi\,T} \log \frac{Q_1}{Q_2} + D\,(Q_2 - Q_1) + F\,(Q_2 - Q_1) \tag{17}$$

For the case of a very deep well where the pipe friction is large compared to the turbulent losses in the aquifer, the term D might be assumed as zero, and in this case an approximate solution for F would be possible. For the case of a well where the pump intake is at or near the screen the pipe losses are very small; in this case the term F may be considered zero, which makes a solution for D possible. For the case where both pipe loss and turbulent loss in the aquifer are substantial, the pipe loss, FQ^2 in equation (16), might be computed from tables of head loss in pipes. The term FQ^2 could also be obtained from field measurements by installing a measuring pipe extending down to the screen.

Application of Theoretical Method

An illustration of the application of equation (16) to test data is given below. The data used are from the test made northeast of Louisville in 1946. The test was made on a 12-inch well screened in the lower 30 feet of an artesian aquifer averaging 67 feet thick and affected by induced infiltration from the Ohio River. For this case the boundary of zero drawdown is a line source located 400 feet from the well. The effective distance to the external boundary as it relates to drawdown at the well is twice the distance to the line source, or r_e = 800 feet; the inside radius of the screen, r_n = 0.45 foot; the transmissibility determined from pumping-test data at observation wells = 120.000 gpd/ft. = 0.185 ft.^{2}/sec.

So far as is known, no solution is available for determining partial-penetration corrections for problems involving both laminar and turbulent flow.

In order to illustrate the use of the theory presented above, the test ta have been converted to the case of full penetration by the application of K ny's[8] approximate equation.

TABLE 2

Test data adjusted for partial penetration

Step	Q (cfs)	s_w a/ (feet)
1	0.819	5.28
2	1.150	7.74
3	1.587	11.33
4	1.961	14.81

a/ One-hour drawdowns equal the sum of the incremental drawdowns caused by each change in pumping rate.

Equation (17) was solved for D by the substitution of test data. Inasmuch as the pump intake was at the top of the screen, the pipe-loss term $F\,(Q_2 - Q_1)$ was considered to be very small and was neglected. Values of D were determined from steps 1 and 2 as 1.73; steps 2 and 3 as 1.57; and steps 3 and 4 as 1.59. The average value of 1.63 sec.^{2}/ft.^{5} was adopted.

Equation (16) was solved for Q_c for each step (again neglecting the pipe-flow term, FQ^2). Values for Q_c are 0.77, 0.70, 0.76, and 0.78; the average value is 0.75 cfs.

Comparison of Theoretical and Empirical Methods

Solution of the same test data (corrected to case of full penetration) by the log-log graphical method according to the empirical equation

$$s = BQ + CQ^n$$

yields the following: B = 6.05 sec./ft.^{2}; n = 2.54; C = 0.538 sec.$^{2.54}$/ft.$^{6.62}$.

Referring to the sketch, figure 2, we may write

$$BQ = \frac{2.3\,Q}{2\,\pi\,T} \log \frac{r_e}{r_B} \tag{18}$$

Solving this equation yields the value of r_B = 0.69 foot.

From equation (9) we may write

$$\frac{r_B}{Q_B} = \frac{r_n}{Q_c} \tag{19}$$

from which Q_B = 1.15 cfs.

The CQ^n term (see figure 2) may also be set equal to its theoretical equivalent:

$$CQ^n = DQ\,(Q - Q_c) - \frac{2.3\,Q}{2\,\pi\,T} \log \frac{Q}{Q_B} + FQ^2 \tag{20}$$

8. Kozeny (Wasserkraft und Wasserwirtschaft, vol. 28, p. 101); equation is quoted by Morris Muskat in The flow of homogeneous fluids through porous media, p. 274, New York, N. Y., McGraw-Hill Book Company, Inc., 1937.

Neglecting t' FQ^2 term as before, we have an expression equating the empirical exp\ ion for "well loss" to its theoretical equivalent. All terms in the equation have been determined. It is found that the equation is not mathematically sound. The trouble may be in the assumption that all the losses between the radius r_B and the well can be expressed empirically as CQ^n; should C or n perhaps be treated as variables? Or, perhaps, the trouble may be in the term D in the equation. This term, the constant for turbulent flow in the aquifer for a given well, neglects the effect of development of the well. Variation in D might be expected as the turbulent zone is expanded through the developed zone and into the undisturbed aquifer.

It must also be recognized that equation (14) was developed on the basis of the assumption that head loss in the turbulent zone varies as the square of the velocity. Since the Reynolds curve shows a transition from laminar to turbulent flow, equation (14) is an over simplification of a complex relationship. An attempt was made to include a more accurate representation for equation (14). Reynolds number times friction factor versus Reynolds number were plotted on log-log paper. A constant was subtracted by trial until a straight line was obtained. The equation of this solution was integrated between r_t and r_n. The resulting expression when introduced in equation (16) complicated the equation to the extent that it became unusable for the problem under consideration.

In order to study the degree of error in the equation, values of D were computed for various discharge rates, using the computed values of all other terms in the equation. Figure 5 shows a plot of the values of D. For values of Q up to 5 cfs, values of D required to make the theoretical and empirical (log-log) methods agree are within a few percent of the average value of 1.63 determined by theory, so that extension of the graphical solution appears to correspond very nearly to theory in this range of discharge for this case. At low rates of Q, that is, at rates just above the critical discharge Q_c, the equation is very sensitive, and values of D relating the theoretical and empirical methods are unreliable. However, in this range the error in total drawdown will be small inasmuch as the magnitude of the "well loss" is small.

Figure 6 shows the variation of drawdown with discharge for the example. Curve A represents the formational head loss outside the turbulent zone, computed from equation (16). Curve B is the total drawdown in the well obtained by adding the theoretical loss in the turbulent zone to curve A. Curve C is the head loss assigned to the formation by the graphical method (BQ), and curve D is the total drawdown in the well by the graphical method $s_w = BQ + CQ^n$. It should be noted that curves B and D agree very closely. This comparison indicates that the empirical graphical method gives a total drawdown consistent with theory, for the case demonstrated, so that the graphical method may be applied to problems of pump design and determination of maximum yield of a given well. The graphical method is not applicable for solution of head distribution outside the well, or for determination of design well radius, as shown by the disagreement of lines A and C.

Distribution of Head Loss in the Aquifer

The distance to the boundary between laminar and turbulent flow may be approximated for various discharge rates from the relation $\frac{r_n}{Q_c} = \frac{r_t}{Q}$ (from equation 10).

In figure 7 are shown profiles of head loss for various rates of discharge, and the extent of the turbulent zone for each rate, for the example used.

The curve of head loss through the turbulent-flow zone is compu' 'rom the equation

$$s_t = DQ^2 r_n \left(\frac{1}{r} - \frac{1}{r_t}\right) \tag{21}$$

Inspection of figure 7 shows the development of the turbulent-flow zone as discharge increases. For discharges less than critical ($Q_c = 0.75$ cfs in this case) the distribution of head loss in the formation follows the normal variation ($\log \frac{1}{r}$). At higher discharges the logarithmic distribution applies outside the boundary between laminar and turbulent flow. In the turbulent zone the head loss changes at a much greater rate. Note that for a discharge of 5 cfs the head loss from the external boundary to the critical zone ($r_t = 3.00$ feet) is 23.9 feet, whereas the loss through the turbulent zone (from $r_t = 3.00$ feet to $r_n = 0.450$ foot) is 34.7 feet. An 18-inch well ($r_n = 0.656$) would reduce the turbulent loss from 34.7 feet to 21.9 feet, and a 24-inch well ($r_n = 0.906$) would have a corresponding turbulent loss of only 14.0 feet.

This diagram, which must be recognized as showing only approximate relationships, indicates that the well radius may be more important in well design than has been considered in the past. Textbooks now in circulation state that the well radius is not very important because the discharge varies as $\log \frac{r_e}{r_n}$, a term that varies little as r_n is changed but that is based on the assumption of laminar flow all the way to the well. This paper demonstrates that the well radius becomes more and more important as discharge increases.

The variation of specific capacity with Q is shown in figure 8 for wells of various diameters at the site of the Louisville test. This figure shows a rapid decline in specific capacity as the discharge is increased beyond the critical discharge. The figure also shows the effect of well radius. For example, at 0.5 cfs the specific capacities of a 12-inch and an 18-inch well are 0.156 and 0.164 cfs/ft., respectively, or a difference of less than 5 percent; at 4 cfs the specific capacities are 0.097 and 0.122 cfs/ft., or a difference of more than 20 percent.

Well Efficiency

In figure 9 are shown "well-efficiency" curves for 12-inch, 18-inch, and 24-inch wells pumped at various rates. For this plotting "well efficiency" is defined as the ratio of (1) the theoretical drawdown computed by assuming that a logarithmic distribution of head is applicable all the way to the well face (in other words, no turbulence is present) to (2) the drawdown in the well. These curves show a rapid drop in efficiency when discharges are increased beyond the critical discharge, and also show the importance of the well radius at higher rates of discharge.

Variations of Drawdown with Time

The foregoing discussion is based on a constant time; that is, all relationships are for a well pumped at a constant rate for a given time (1 hour in the example). In the example recharge from the river was occurring, and steady-flow conditions existed.

For the case of a well in an infinite aquifer with neither barriers nor recharge, the expression for drawdown in the well $s_w = BQ + CQ^n$ may be altered[9] to include the time factor by equating the formational term BQ to the drawdown as expressed by the simplified Theis equation

$$s_w = \frac{Q}{4 \pi T} (2.3 \log \frac{4Tt}{r_w^2 S} - .577) + CQ^n \qquad (22)$$

in which S is the storage coefficient and r_w the "effective radius" of the well.

If we use Jacob's definition for effective radius, "the distance, measured radially from the well, at which the theoretical drawdown based on the logarithmic head distribution equals the actual drawdown just outside the screen," the value of r_w is unknown.

Reference to figure 2 and to the comparison of the empirical and theoretical methods of analysis shows that the distance from the well center to the point where the BQ and CQ^n portions of the drawdown are separated (r_B) occurs at the critical-flow boundary for a discharge Q_B. In the example the term r_B equals 0.69 foot. This seems a logical radius to use in the time equation in place of r_w, since losses beyond this point are expressed empirically as formational losses, and losses between r_B and r_n as "well losses" which are considered constant with time.

Although Jacob did not mention partial penetration, it is obvious that the equation must be altered if the pumped well penetrates only a part of the aquifer.

If the empirical expression is accepted, then the "well loss" CQ^n hypothetically occurs in a very narrow zone (between $r_B = 0.69$ foot and $r_n = 0.45$ foot). If r_B be considered the "effective radius" of the well the penetration factor would be applied only to the BQ term, using r_B in place of r_w in the Kozeny equation. If it be assumed that penetration effects are present between r_B and r_n, then the correction applies to both BQ and CQ^n, and r_n should be used in computing the correction. It should be noted that the Kozeny equation was derived for laminar flow and for a condition of steady flow. If the Kozeny equation is used for time problems, it is found that the BQ term is made up of two parts, a constant representing the penetration correction and a variable representing the formational loss which increases with time.

The theoretical drawdown distribution is considerably different than the $BQ + CQ^n$ empirical distribution. Until further analysis is made of the penetration problem, any attempts to relate step-test results to time problems for partially penetrating wells should be carried out with caution.

The empirical expression for drawdown in the well ($BQ + CQ^n$) is applicable to all types of problems for fully penetrating wells and for constant-time problems for partially penetrating wells, and it can be used within certain limits for problems involving partial penetration and time effects.

SUMMARY AND CONCLUSIONS

The important points covered in this paper may be summarized as follows:

1. On the basis of field data for several tests, it has been demonstrated that the empirical equation

$$s_w = BQ + CQ^n$$

9. Jacob, C. E., Drawdown test to determine effective radius of artesian well: Am. Soc. Civil Eng. Trans., vol. 112, paper no. 2321, p. 1055, 1947.

defines the total drawdown of a pumped well more closely than the equation

$$s_w = BQ + CQ^2$$

proposed by Jacob.

2. A graphical method is presented for solving step tests according to the empirical equation $s_w = BQ + CQ^n$ by a simple plotting on log-log paper.

3. Computation of Reynolds numbers shows that, outside the well, laminar flow may occur at low pumping rates and turbulent flow at higher rates.

4. Analysis shows that the boundary between laminar and turbulent flow moves outward from the well face as discharge rates are increased.

5. An approximate equation has been written that evaluates the drawdown in the well in terms of laminar flow in the aquifer outside the critical radius. Turbulent flow from the critical radius to and through the well face, and frictional loss due to upward flow in the well.

6. Comparison of the log-log graphical method and the theoretical method is made as follows:

The $BQ + CQ^n$ method is very simple in application and allows for experimental errors. The exponent n is empirical and should not be confused with the theoretical (square) relationship of turbulent flow. The method is not applicable to problems of head distribution outside the well or for designing radii of wells. The term BQ carries the logarithmic distribution of head into the turbulent zone, so that the separation of the terms for aquifer loss and "well loss" is empirical and does not agree with theory. The approximate equation more nearly describes the true head distribution outside the well and should be used for this type of problem.

Comparison of the total drawdown obtained by the two methods shows close agreement at low and medium discharge rates but some deviation at higher rates. The deviation may be the result of erroneous assumptions or, an erroneous assumption in the empirical expression or may result from factors such as well development, not included in the theoretical analysis. In view of the fact that the deviation in total drawdown computed by the two methods is small at low and medium discharge rates and that uncertainty exists as to which method is in error, it is suggested that the empirical method be used because of its simplicity.

7. The bearing of radius on well design is more important than is indicated in current literature. Under certain conditions large savings in head loss can be made by using larger-diameter wells.

8. Efficiency of a well falls off rapidly as discharge is increased.

9. For fully penetrating wells step-test data may be used in problems having a variable time factor. For partially penetrating wells, the step-test results are satisfactory for constant-time problems, and can be used for variable-time problems under certain conditions.

10. The entire problem of effects of partial penetration should be investigated.

11. Other factors, such as the effects of development of a well, should also be investigated.

APPENDIX

Notation

B Head loss of formation per unit of discharge

C "Well-loss" constant for turbulent flow for a given well

C	" -loss" constant for laminar flow for a given well
D	Aquifer constant for turbulent flow for a given well
E	Aquifer constant for turbulent flow for any well
F	Constant for head loss due to turbulent flow up the well
M	Head loss between the external boundary, r_e, and the inside of the screen, r_n, according to logarithmic distribution for laminar flow
N	Head loss between the turbulent-laminar boundary (r_t) and the inside of the screen (r_n) according to logarithmic distribution for laminar flow
Q	Discharge of well
Q_B	Discharge at which the actual turbulent-laminar flow boundary coincides with the empirical boundary r_B
Q_c	Critical discharge below which laminar flow prevails
S	Coefficient of storage
T	Transmissibility of aquifer; a property of the aquifer expressed as the quantity of water flowing through a vertical section of the aquifer of unit width, under a gradient of unity
a	Area
m	Thickness of aquifer
n	Unknown constant power relating discharge to "well loss"
r_1, r_2	Distance from well center to any point 1, 2, --
r_e	Effective distance from well center to boundary of zero drawdown
r_B	Radius at which turbulent and laminar losses are separated by the empirical equation $s_w = BQ + CQ^n$
r_n	Inside radius of well or well screen
r_t	Distance from well center to boundary between laminar and turbulent flow
r_w	"Effective radius" of well
dr	Increment of distance from well center
s	Drawdown at any point caused by pumping a well
s_w	Drawdown in the pumped well; at the end of any multiple step, s_w equals the sum of the incremental drawdowns for all steps
s_t	Head loss due to turbulent flow between the turbulent-laminar boundary (r_t) and the inside of the screen (r_n)
s_f	Head loss inside the well resulting from upward flow in the casing
s_1, s_2	Drawdowns at any point a distance r_1, r_2, -- from well center
$\Delta s_1, \Delta s_2$	Increments of drawdown resulting from changes in pumping rate
ds	Increment of drawdown
log	All logarithms are to the base 10
v	Velocity
θ	Porosity

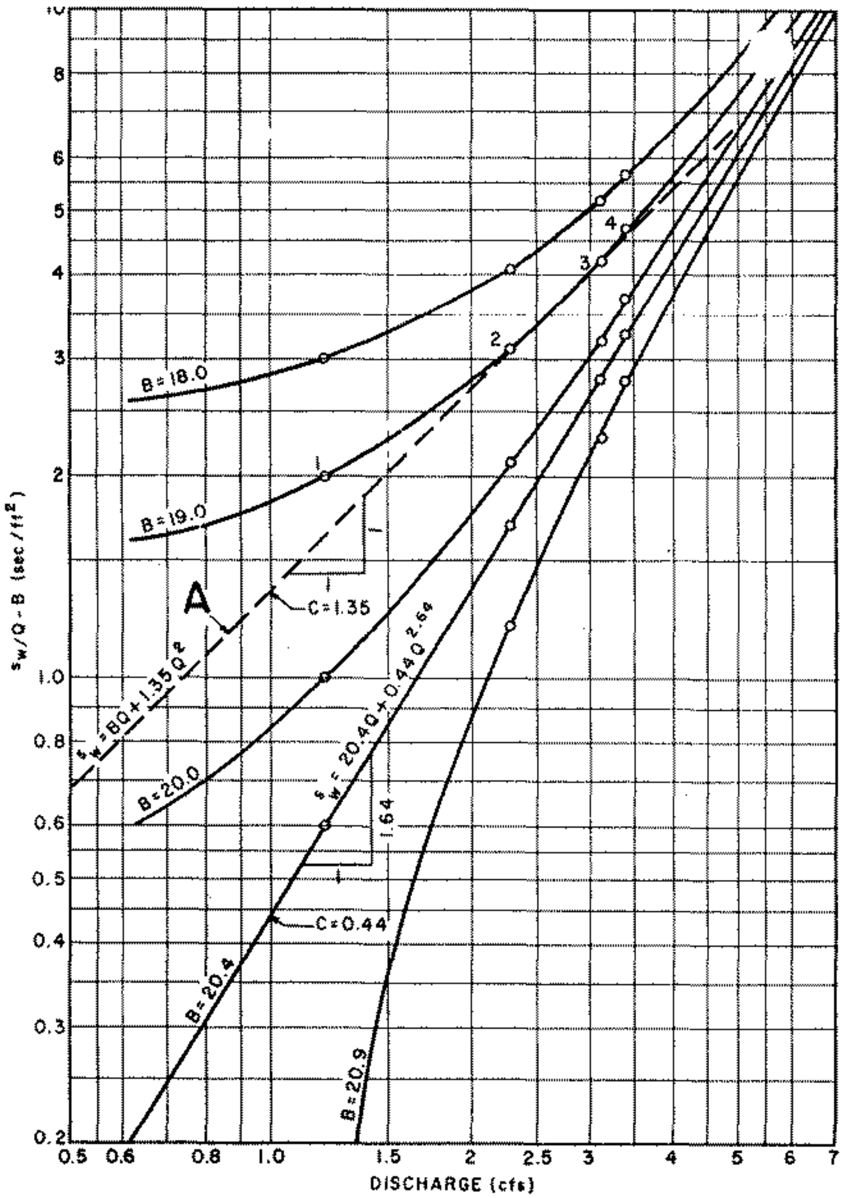

Figure 1.--Graphical solution of step test ($s_w = BQ + CQ^n$) and comparison with Jacob's solution ($s_w = BQ + CQ^2$). Data from Long Island test used by Jacob.

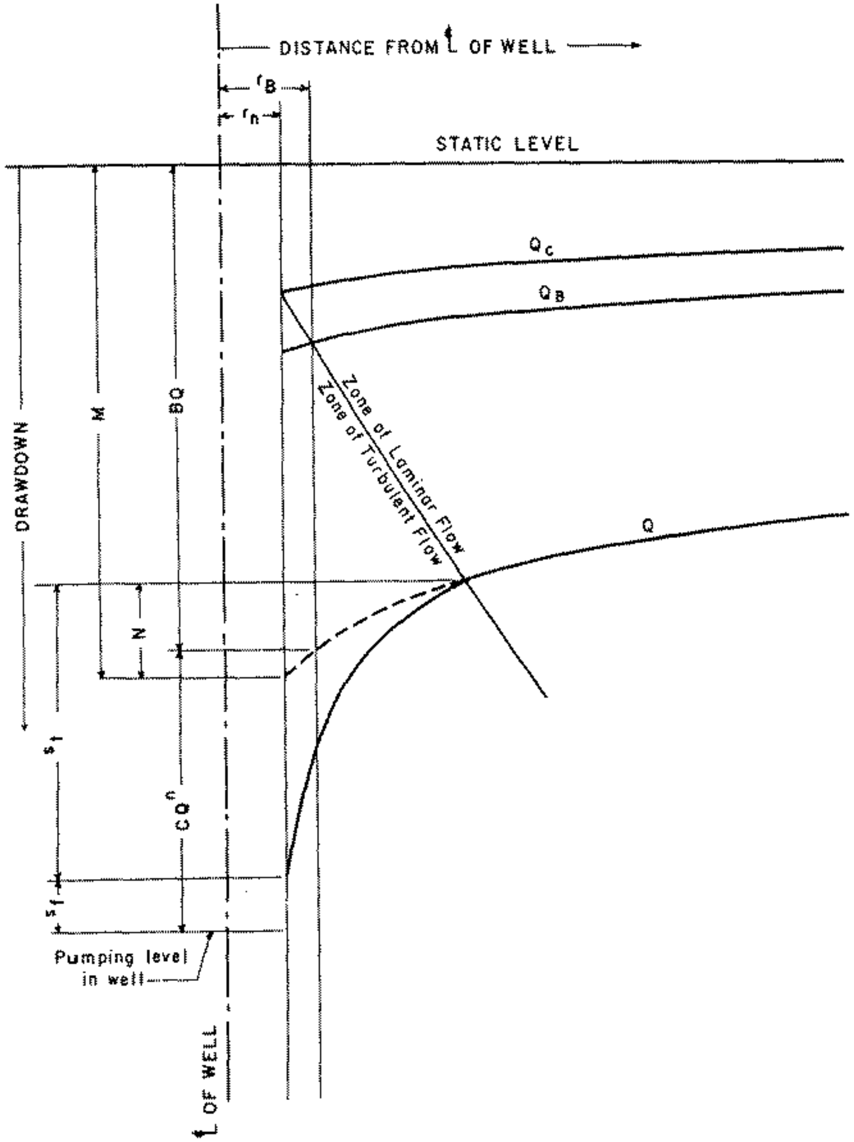

Figure 2. --Generalized section showing head distribution in and near a pumped well

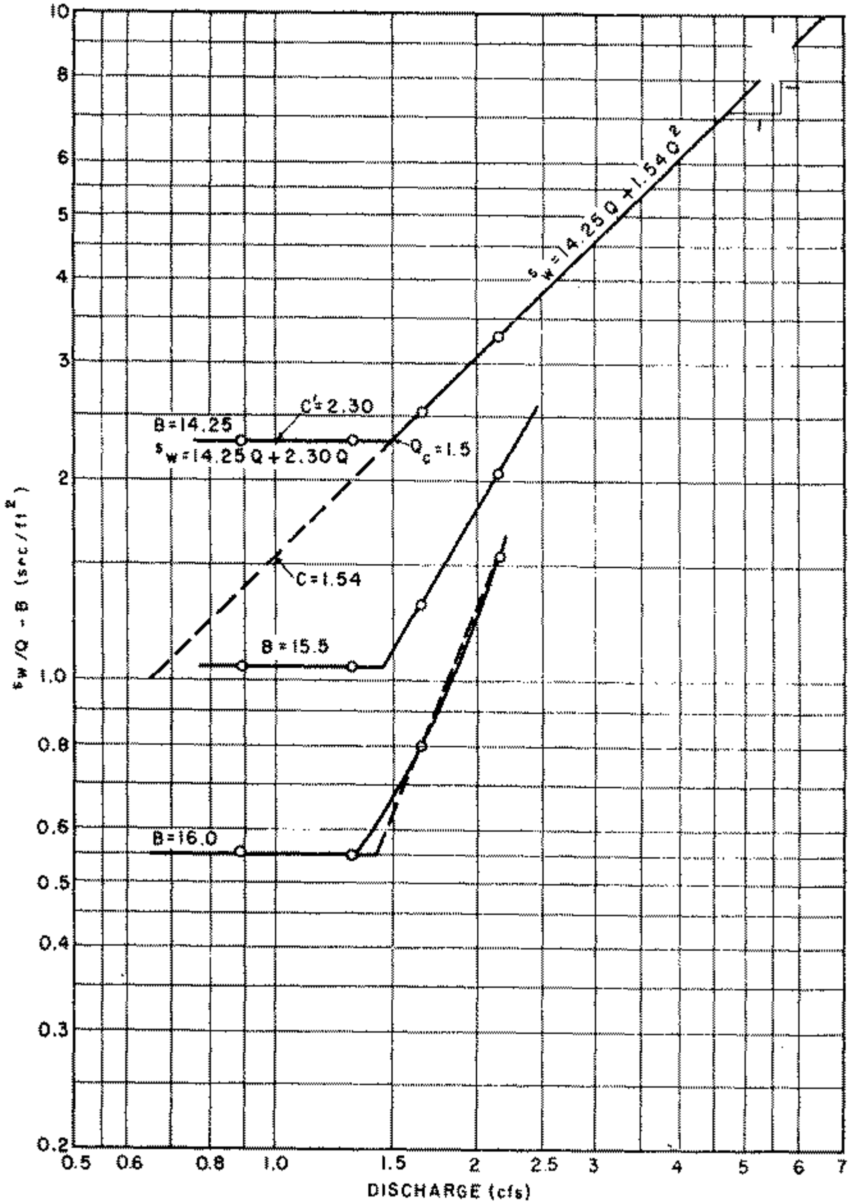

Figure 3. --Graphical solution of step test run in 1945, southwest of Louisville, showing discharges above and below critical discharge

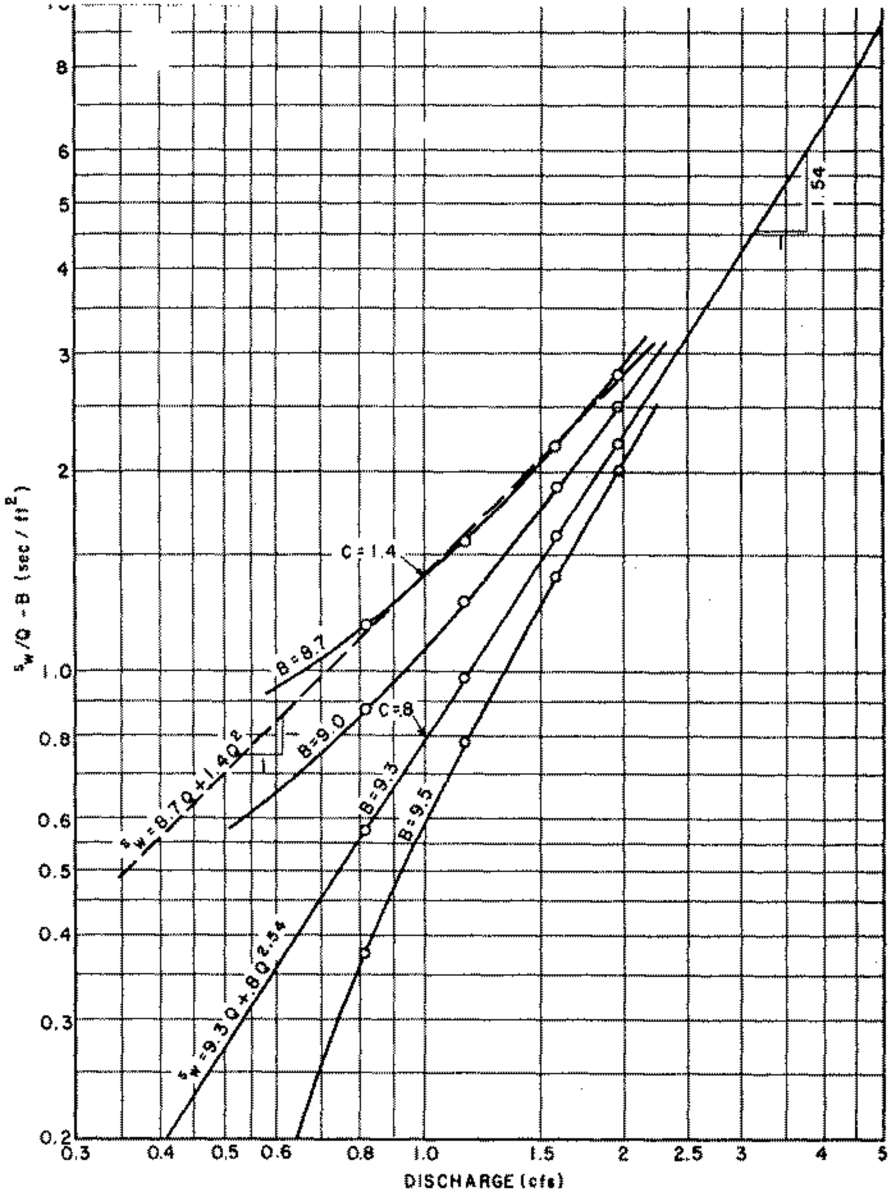

Figure 4. --Graphical solution of step test run in 1946, northeast of Louisville

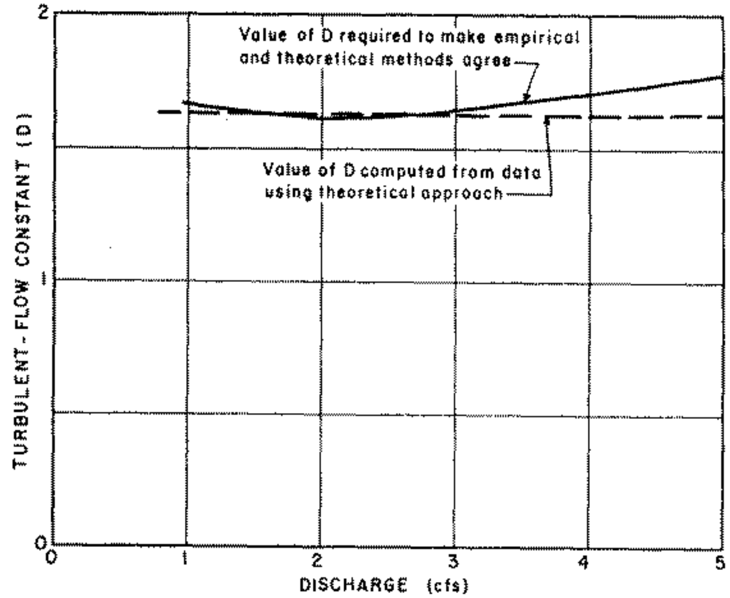

Figure 5. --Graph showing variation in turbulent-flow constant, D, required to make theoretical and empirical methods agree

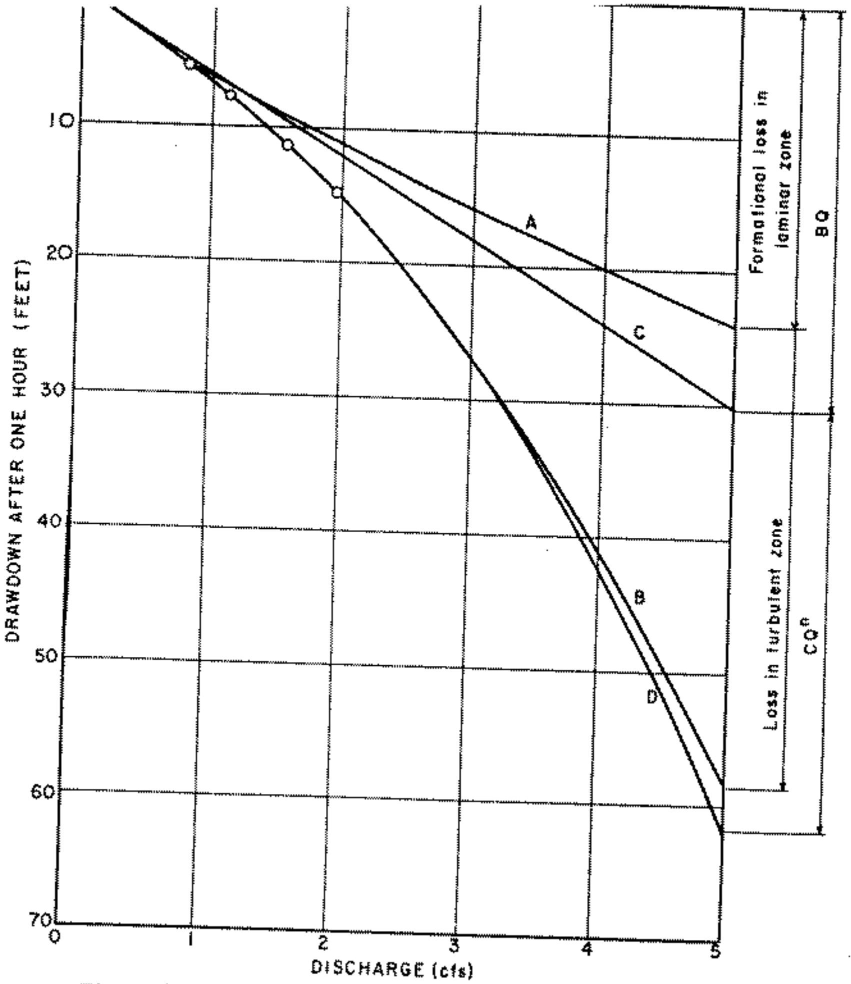

Figure 6.--Graph showing 1-hour discharge-drawdown relationship as computed by theoretical (B) and empirical (D) methods (northeast Louisville test, 1946; adjusted to full penetration); also shows proportion of head loss assigned to laminar and turbulent flow

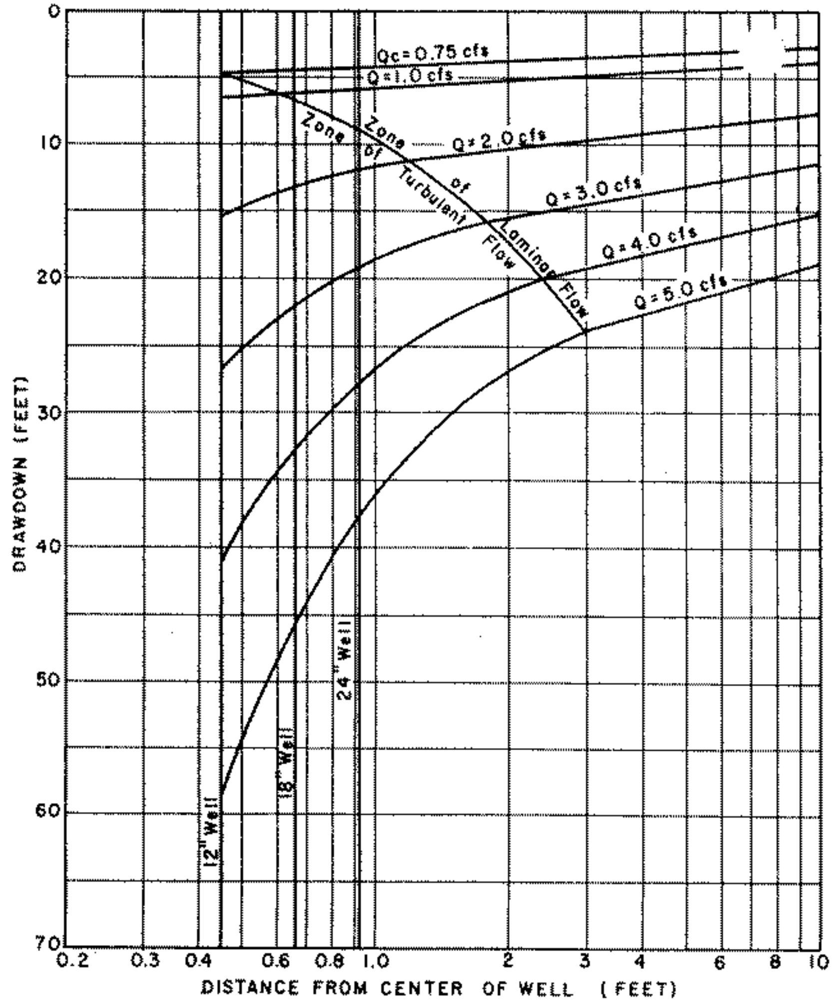

Figure 7.--Graph showing 1-hour head distribution near a pumped well for various pumping rates, (data from northeast Louisville test, 1946, adjusted to full penetration)

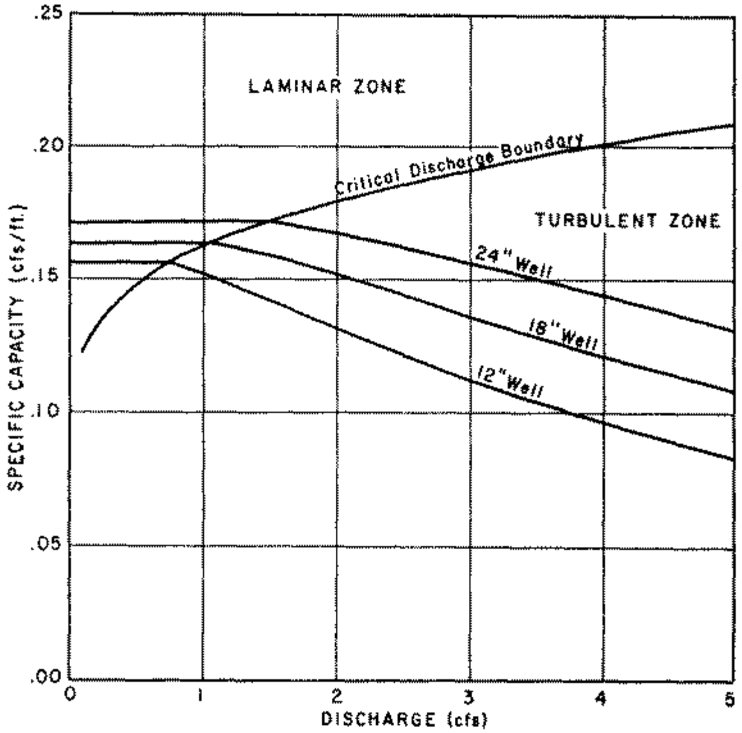

Figure 8.--Graph showing variation of 1-hour specific capacity with discharge and with well radius (northeast Louisville test, 1946, adjusted to full penetration)

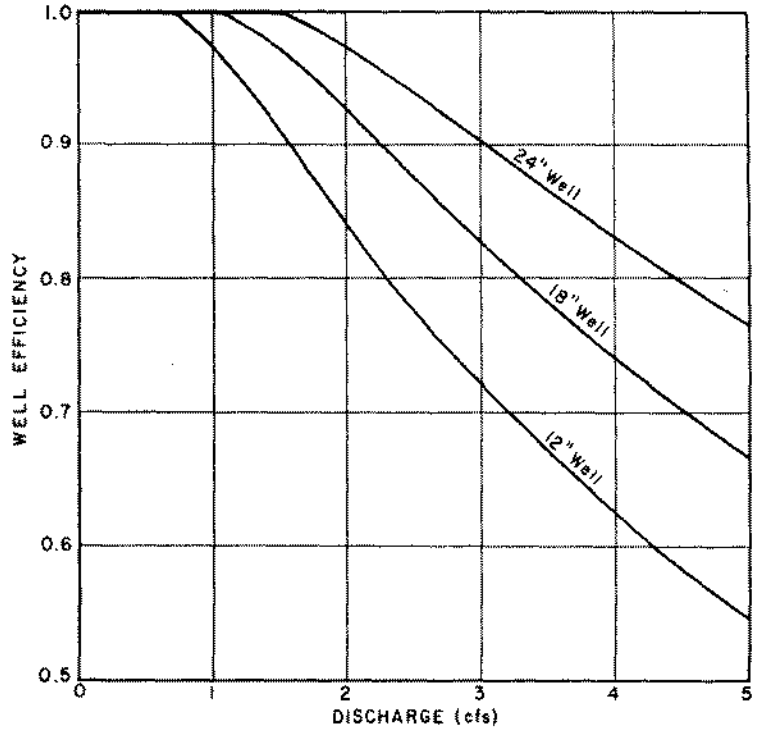

Figure 9.--Graph showing 1-hour well efficiency for wells of various diameters and various discharges (northeast Louisville test)

DETERMINING WELL EFFICIENCY BY MULTIPLE STEP-DRAWDOWN TESTS

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Résumé

La qualité d'un puits peut être évaluée par analyse graphique de l'équation approchée :

$$S_w = BQ + CQ^2$$

dans laquelle :

— S_w est le rabattement total dans le puits pour le débit Q.

— BQ représente le rabattement provoqué par la perte de charge de la formation aquifère.

— CQ^2 représente le rabattement provoqué par la perte de charche propre au puits.

Le rendement du puits peut être défini par la valeur du rapport BQ/S_w.

Le rendement d'un puits dépend largement de l'importance de la perte de charge propre au puits et par conséquent diminue très rapidement avec l'augmentation du débit. Le rendement d'un puits dans une formation aquifère ayant un haut coefficient de perméabilité est affecté par la perte de charge propre au puits à un plus grand degré que le rendement d'un puits dans une formation de faible perméabilité, et est moins affecté par une pénétration partielle dans des aquifères de grand coefficient de perméabilité.

Pour un puits foré à un emplacement déterminé, on peut atteindre le rendement optimum en crépinant sur une épaisseur d'aquifère aussi grande que possible, avec une surface de perforation aussi grande que le permet la granulométrie naturelle de la formation. Cette surface perforée peut être augmentée en utilisant une crépine plus longue, une crépine de plus grand diamètre, ou une crépine ayant un coefficient de perforation plus élevé par rapport à la surface totale de la crépine. Le rendement maximum ne peut être obtenu que si le développement du puit sa été effectué à un degré suffisant pour enlever les matériaux fins situés contre la face externe du puits et dans les formations adjacentes de manière à créer un filtre stable de gravier naturel ayant pour effet une perméabilité plus grande au voisinage du puits. De cette façon non seulement on diminue la vitesse d'arrivée de l'eau mais on empêche aussi l'entrée de matériaux fins dans le puits et le pompage de sable.

L'efficacité du développement peut être appréciée en comparant la perte de charge propre au puits à la perte de charge de la formation aquifère. Le degré de détérioration d'un vieux puits peut être apprécié par comparaison des valeurs de la perte de charge propre au puits, ou de préférence en comparant les résultats de tests effectués au moment de l'achèvement du puits et après que le puits a été en service pendant une certaine durée de temps.

En accumulant et en interprétant les données empiriques obtenues par le procédé des essais de rabattements on devrait pouvoir apporter une contribution importante à la compréhension des caractéristiques de qualité des puits, la part étant faite du comportement variable des aquifères. L'appréciation des facteurs affectant le rendement des puits est le seul moyen d'arriver à des économies sur les coûts de construction et les frais d'exploitation.

Abstract

Graphical solution of multiple step-drawdown test data permits an approximate determination of the two components of drawdown in a pumped well; that due to formation loss, and that due to well loss. This in turn permits an engineering estimate of the "efficiency" of a well.

An understanding of the various factors affecting well efficiency is important because savings in well design and construction and operation can be made by increasing the efficiency of a well and thus preventing much needless waste of materials and pumping energy.

Data collected during step-drawdown tests of 32 screened wells tapping alluvial sediments in northern and western Iran are presented, together with data from 16 tests on 11 perforated and 3 screened wells in southeastern Washington State.

1. Introduction

The purpose of ground-water development is to bring about an additional supply of water, whether it be used for agricultural, domestic, industrial, or other purposes. The construction of a water well provides the means for tapping and exploiting the underground reservoir, and the real cost of this well is determined more often than not by pump repairs, maintenance, and pumping costs than by the original investment in materials, drilling, and development.

The step-drawdown test provides a means whereby at any time during the life of a well it should be possible to determine with considerable accuracy the two components of its specific drawdown (i.e. those attributable to formation loss and to well loss). This determination in turn should facilitate an appreciation of the "efficiency" of a well and any change in such due to encrustation or clogging of the screen or sand packing of the gravel wall. Through the accumulation of data in a given area, as wells are developed and placed in operation, such tests should greatly assist in the selection of screen type, length, slot opening, and gravel size — and even the determination of whether or not an artificial gravel pack is required. Much guess work can thus be eliminated and the efficiency of wells increased with the consequent prevention of much needless waste of pumping energy and the proper selection of pump and motor to give optimum performance throughout the life of the well.

This paper describes a procedure for evaluating the data of a multiple step-drawdown test. Because the amassing of empirical values of well-loss constants together with pertinent data on the design and construction of wells may assist others in further appraising the various factors defining the performance of a well, information is presented in tabular form for 47 tests. These data permit comparisons to be made between wells in alluvial aquifers of different permeabilities, between developed and undeveloped wells, and between screened and perforated wells. The data also permit a relative evaluation of the efficiency of production wells at required pumping rates and suggest a method for appraising the effectiveness of well development.

2. Summary of Geohydrologic conditions

The step-drawdown tests reported herein were conducted on wells tapping fluviatile deposits of Quaternary Age. Of the 47 tests, 31 were run on production wells located in northern and western Iran (see Figure 1). The alluvial sediments were penetrated to depths ranging from 50 feet to 330 feet; the material ranging in size from clay to boulders. The groundwater occurred under both water-table and artesian conditions depending on local geologic controls.

Sixteen tests were run on observation wells located in the southeastern part of the State of Washington. Here the saturated sediments consisted of glaciofluviatile sands and gravels ranging from 30-50 feet in thickness, and of 15-220 feet of lacustrine deposits ranging in size from clay through cobbles with the finer fractions predominating. In general, the groundwater occurred under water-table conditions although locally artesian conditions existed due to the existence of cemented deposits or thick clay lenses.

Semi-arid climatic conditions prevailed both in Iran and in southeastern Washington State.

3. Previous Work

The step-drawdown technique has received relatively little professional recognition and only a few papers have been widely published. Hence a brief summary of pertinent papers is justified and appropriate.

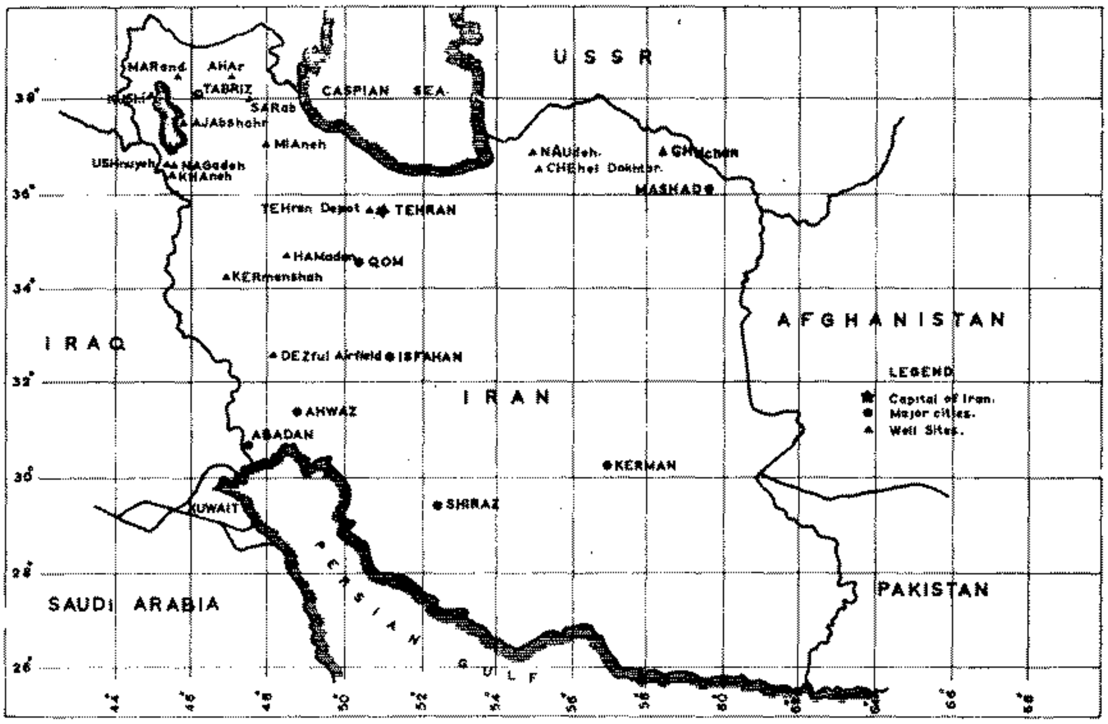

Fig. 1 — Map of Iran showing locations of well sites.

According to Jacob's 1946 paper, (1) the drawdown in a well that is pumped has two components : the first, arising from the "resistance" of the waterbearing formation (formation loss), is proportional to the discharge ; and the second, termed "well loss" and representing the loss of head that accompanies the flow through the screen or perforations and upward inside the casing to the pump intake, is proportional approximately to the square of the discharge. The resistance of an extensive formation increases with time as the ever-widening area of influence of the well expands. Consequently, the specific capacity of the well, which is discharge per unit drawdown, decreases both with time and with discharge.

In 1953 Rorabaugh (2) presented the empirical equation $s_w = BQ + CQ^n$ which defined the total drawdown of a pumped well more closely than the equation $s_w = BQ + CQ^2$ proposed by Jacob ; in each case s_w is the drawdown in the pumped well ; B, the formation constant ; C, the "well loss" constant ; and Q, the discharge.

Bruin and Hudson (3) in 1955 opined that Rorabaugh presented the more exact method despite the complication of having to evaluate the three terms B, C, and n. They concluded that for practical engineering application, Jacob's equation was the more useful.

4. Example of Analysis

The example chosen to illustrate the approximate analysis of the stepdrawdown test is a pumping test of a production well located at Naudeh, in northern Iran.

Well NAU-3 was pumped at five rates, 100, 200, 400, 500, and 550 gallons per minute (gpm), and each rate was maintained for two hours. The first step of the analysis (3) was to plot the test data on semi-logarithmic graph paper with the drawdown on the arithmetic axis and the elapsed time after pumping began on the logarithmic axis. See Figure 2. It can be noted that the recession curve at 100 gpm had a "slope" of 0.30 feet per log cycle. The slopes at higher rates were estimated as shown on the figure. These slopes were used to extrapolate each step of the test beyond the period of pumping of each step as shown by the dashed lines in Figure 2. These extrapolations were used to obtain the incremental drawdown caused by a change in pumping rate. The two-hour incremental drawdowns for stated rates are shown in tabular form on Figure 2 as are

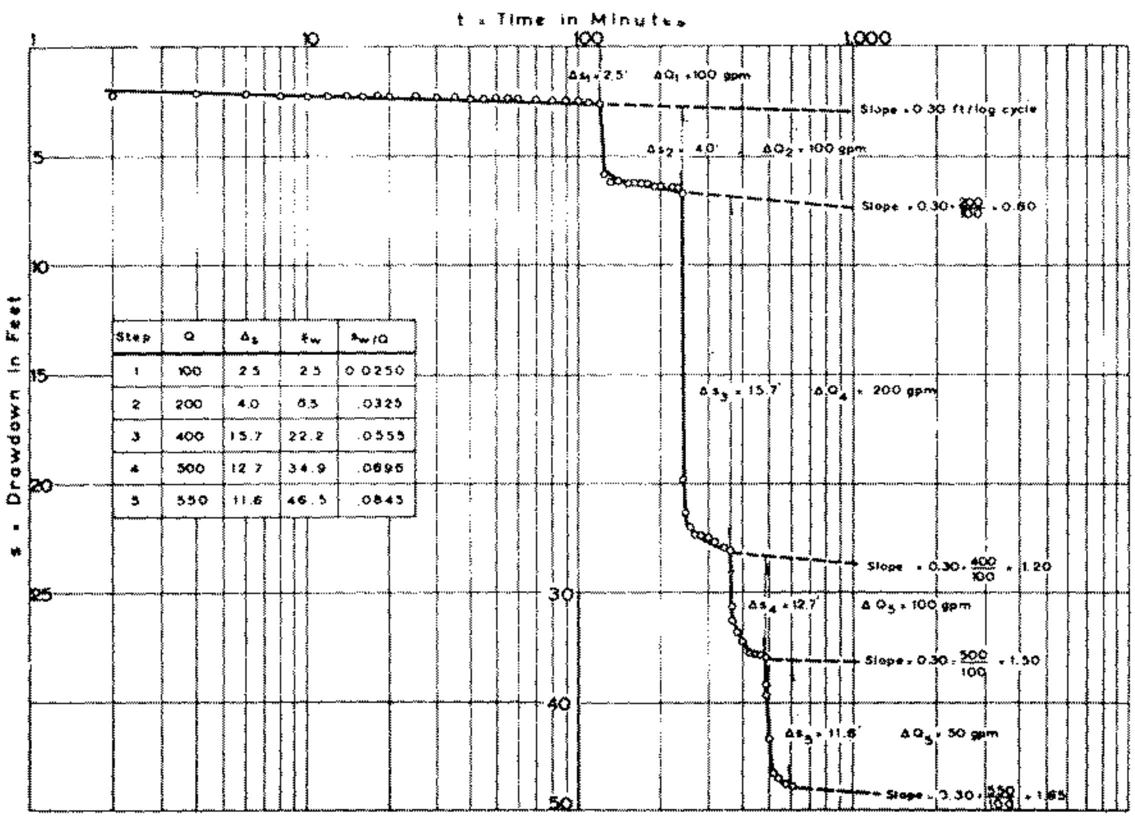

Fig. 2 — Well Nau - 3, Step — drawdown test curves.

the values of s_w and s_w/Q. The values of s_w and s_w/Q were plotted on arithmetic coordinate paper as shown in Figure 3 and the straight-line approximation through the points was extended back to 0 gpm. The equation of the form $s_w/Q = B + CQ$ fits this line. The value of B is the value of the intercept of the line with the s_w/Q axis and the value of C is the slope of the line. The equation $s_w/Q = 0.012 + 0.00012Q$ was determined from which $s_w = 0.012Q + 0.00012Q^2$ which is the form of Jacob's equation (1) and is the approximate equation for the drawdown in Well NAU-3 for a pumping period of two hours. Figure 4 shows a plot of this equation and the observed drawdowns for the five pumping rates.

Well NAU-3 was designed to produce 100 gpm for 16 hours daily. The observed drawdown after two hours of pumping at 100 gpm was 2.55 feet, whereas the theoretical drawdown is calculated to be 2.40 feet; half the drawdown being attributable to formation loss and the other half due to well loss. Figure 4 also shows the drawdown-yield curve for Well NAU-2 which is located 480 feet northeast of NAU-3 and for which the drawdown equation was estimated to be $s_w = 0.08Q + 0.00074Q^2$. A comparison of the two curves readily shows that Well NAU-3 is the superior of the two.

Drawdown equations for longer pumping periods may be determined in the same way as discussed above. The values of C should not be affected by time, but B should be expected to vary with the logarithm of time.

5. Tabulation of data

Well data and the results of 47 pumping tests are given in Table 1. The table is divided into four sections : the first lists wells which tap aquifers with a range in field permeability from 85-700 gpd/ft^2 ; the second, wells tapping aquifers with a permeability range 840-1,700 gpd/ft^2 ; the third, permeability range 2,000-8,700 gpd/ft^2 ; and the fourth, permeability ranging above 10,000 gpd/ft^2.

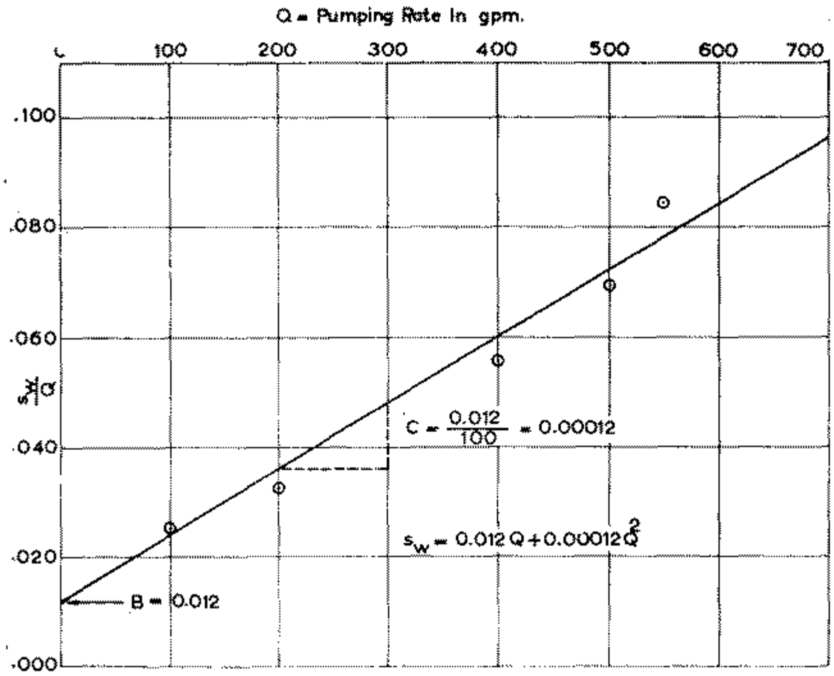

Fig. 3 — Plot of s_w/Q *vs.* Q to solve for values of B. and C.

Wells with a 3-letter prefix are located in Iran (see Fig. 1), and those without the prefix are located in a 500 square mile area in southeastern Washington State. Eleven wells in Washington State are 8-inch diameter cased wells with perforations cut with a Mills knife perforator ; three are 12-inch screened wells. All the 31 wells in Iran were completed with 12-inch Johnson stainless steel well screens. The wells in Washington State are observation and monitoring wells and were not designed for production purposes. The wells in Iran were constructed as domestic water-supply wells, and thus screens were designed to permit entrance of the 50-70 per cent finer by weight sample of the water-bearing formation. The lenght of the screens was usually governed by the Client for economic reasons after due consideration was given to site requirements and aquifer thickness.

None of the perforated wells were ever developed prior to testing. During pumping tests (with a 6-inch Layne-Bowler 4-stage submersible pump) considerable quantities of fine to coarse sand were pumped as the formation flowed through the large perforations. The pumping life of the test pump was less than 500 hours, and twice the impellers had to be replaced because of cavitation due to sand pumping. The three screened wells in Washington State were developed with a surge block for 1-1½ days only.

In Iran all the wells were developed until sand-free water was produced on test for at least six hours. The wells were developed for short periods using a surge block, then for a greater period by surging and pumping with compressed air using either a 315 cfm or a 600 cfm compressor, and finally for a short period with a test pump (10-inch Worthington 6-stage turbine) that overpumped and backwashed the well.

The tabulation of pumping test results includes the results of step-drawdown tests analyzed as in the example above, and the results of drawdown-recovery tests analyzed by one or more of the graphical methods in common use ([4, 5]). The data thus presented give values of the formation and well-loss constants of a pumped well and values of transmissibility and permeability of the tapped aquifer.

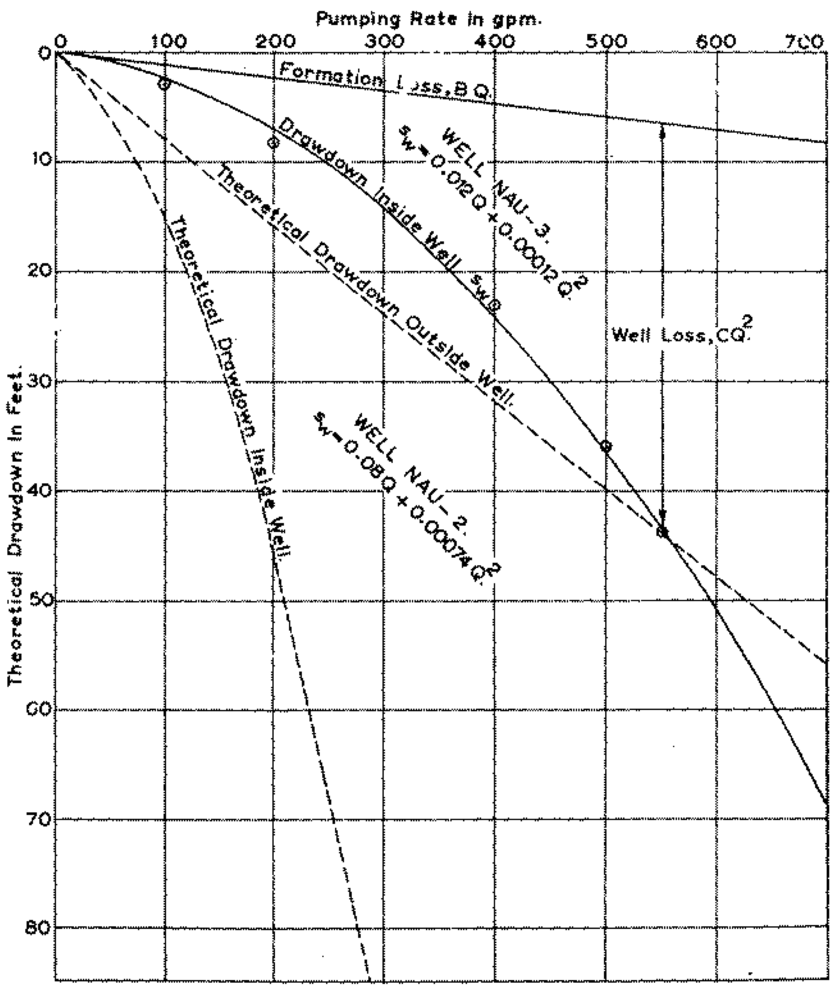

Fig. 4 — Drawdown-Yield Curves.

6. Efficiency of wells

Rorabaugh ([2]) defined "well efficiency" as the ratio of (a) the theoretical drawdown computed by assuming that no turbulence is present (or essentially, BQ) to (b) the drawdown in the well, s_w. Walton ([6]) defines the efficiency of a well as the ratio of the theoretical specific capacity to the actual specific capacity of the well. Factors influencing the actual specific capacity include the hydraulic properties of the aquifer (coefficients of transmissibility and storage), geohydrologic boundaries of the aquifer, the partial or total penetration of the aquifer, the effective open area of the well screen or perforated casing, duration of pumping, and pumping rate. Rorabaugh ([2]) recognized that factors such as well development, not included in theoretical analyses, are important and affect the predictions as to total drawdown and consequent estimates of well efficiency. Thus, the efficiency of a well depends also upon construction features and development of the well — two factors extremely difficult to evaluate.

In Figures 5 through 8 are shown "well-efficiency" curves for the tested wells at various theoretical pumping rates, or plots of BQ/s_w vs. Q. Each figure includes curves for those wells ending in formations with permeabilities in the same order of magnitude as distinguished in Table 1. These curves immediately show a drop in efficiency when discharge is increased. It follows also that a high value for the well-loss constant, C, and a low pumping rate can result in a low well loss (CQ^2) and a high efficiency. Conversely, a low C and a high Q can result in a higher well loss and a lower efficiency. Furthermore, it will be noted that the efficiency of a well in an aquifer having a high transmissibility is affected by well loss to a greater degree than the efficiency of a well in an aquifer having a low transmissibility inasmuch as formation loss, or BQ, varies inversely with T. If T is low, BQ theoretically is great and well loss is a small proportion of the total drawdown in a well; and if T is high, BQ is small and well loss is a large proportion of the actua l drawdown.

6.1. *Well Construction Factor*

It is apparent after a brief glance at Figures 5-8 that some of the perforated wells are more "efficient" than some of the screened wells and some are less efficient. Because each well was designed, contructed, and completed for specific reasons in different areas under varying conditions, no reasonable direct comparison is possible. It is reasonable to state, however, *all other factors being equal*, a screened well will be more efficient than a perforated well due to a greater effective open area permitting easier ingress and less resistance to flow.

Unfortunately, only one example is available for fair comparison : Wells 55-51 and 55-52 which are located 25 feet apart and both of which are open to and penetrate fully the 45 feet of aquifer. Perforated Well 55-51 has an estimated total of 1,620 square inches of effective open area whereas screened Well 55-52 has 7,640 square inches (Table 1). Figure 8 clearly demonstrates that the screened well has the greater efficiency over the indicated range of pumping rates.

Two examples are available to show that increasing the amount of open area *of a given well* will increase the efficiency. Well S8-19 was completed in August 1950 and the lower 15 feet of casing was perforated 4 perforations per round and 1 round per foot ([7]). The well-loss constant determined in June 1958 was of a value $C = 0.0022$. Immediately after this test the well was gun perforated over an 8-foot section, 4 perforations per round and 4 rounds per foot. A subsequent test gave $C = 0.000062$ (Table 1). Figure 5 shows clearly the great increase in efficiency.

Well 26-15 was first tested in July 1958 to give $C = 0.000090$ and then reperforated using a shaped-charged gun perforator ([7]) and retested to give $C = 0.000012$; Figure 6 shows the increased efficiency obtained after reperforation.

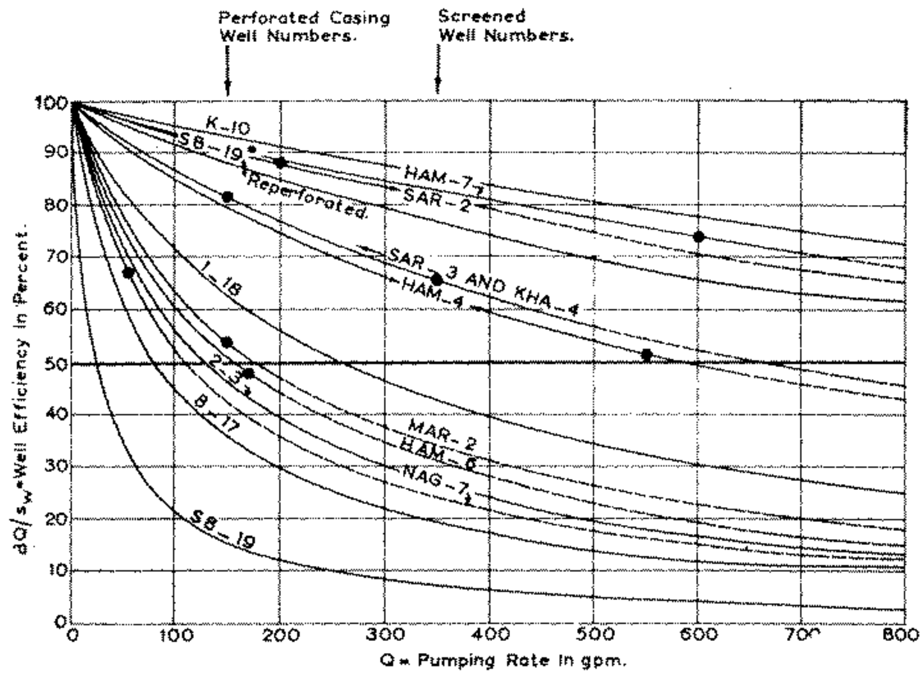

Fig. 5 — Well efficiency curves, permeability 85-700 gpd/ft^2.

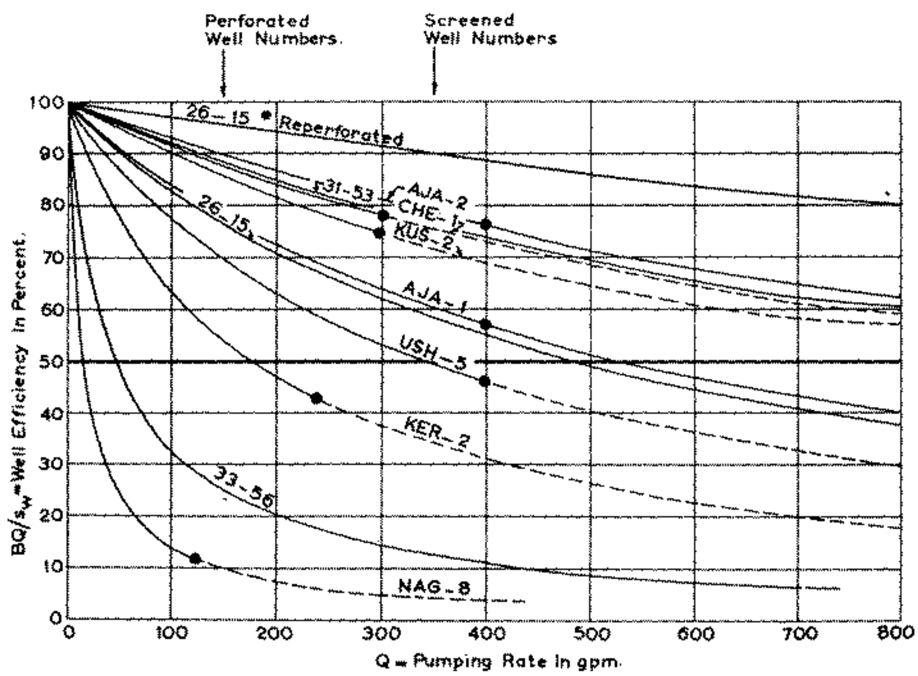

Fig. 6 — Well efficiency curves, permeabillity 840-1,700 gpd/ft^2.

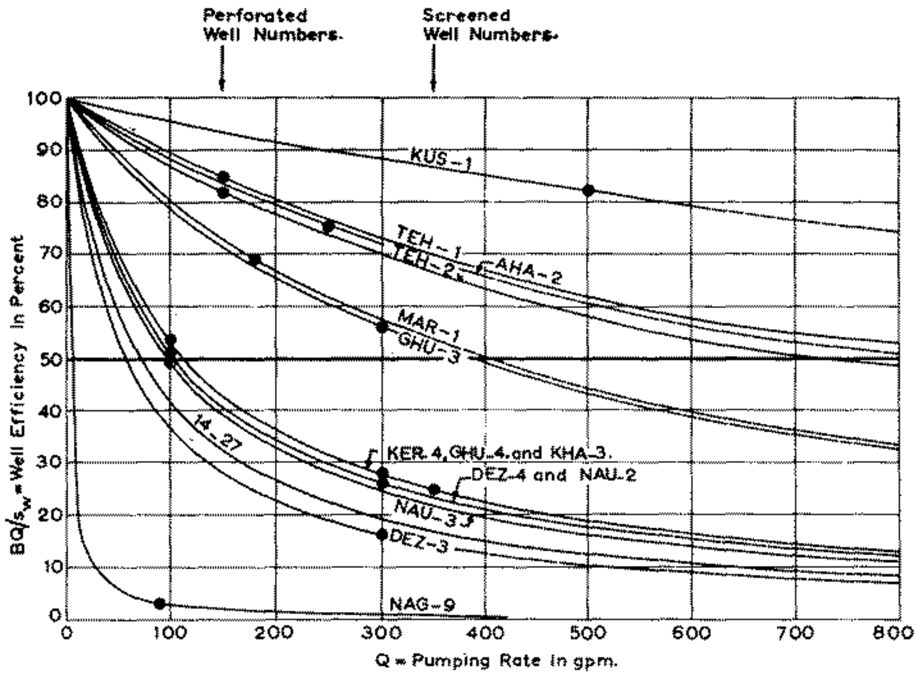

Fig. 7 — Well efficiency curves, permeability 2,000-8,700 gpd/ft^2.

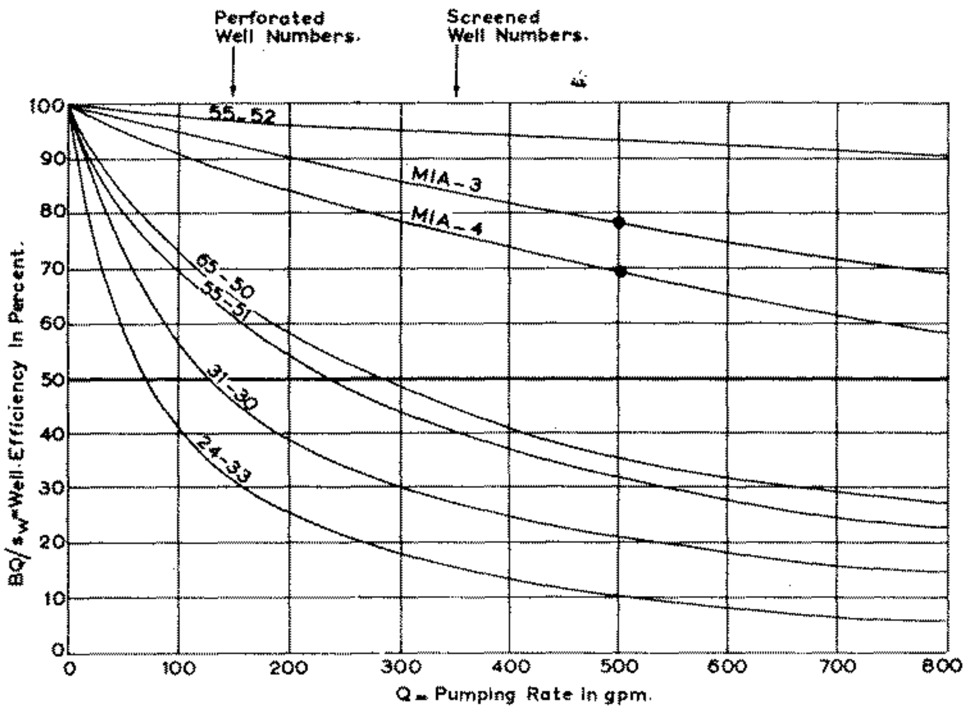

Fig. 8 — Well efficiency curves, permeability > 10,000 gpd/ft^2.

Wells DEZ-3 and DEZ-4 are offered as an example that by increasing the length of screen (and the open area) in a given aquifer, the well-loss constant will be decreased with an accompanying increase in efficiency. Thus, the 15 feet of No. 100 slot screen in DEZ-3 contributed to a well-loss constant value of $C = 0.00017$, and the 20 feet of No. 100 slot screen in DEZ-4 contributed to a lesser constant of $C = 0.00009$. Figures 7 illustrates the difference in efficiency between the two wells.

The efficiency and the specific capacity of a well may also be increased by increasing the radius of the well and by increasing the per cent penetration of the total saturated thickness of the aquifer. As a general rule it has been found that the specific capacity is not greatly increased by increasing the radius of a well because discharge varies as a logarithm of well radius. An increase of about 5 per cent may be expected by increasing the effective radius from 12 to 18 inches, and to only 10 per cent by going to a 30-inchradius. Rorabaugh ([2]), however, demonstrated that the radius becomes more and more important as discharge increases. Thus, the difference of 5 per cent given above increases to more than 20 per cent when discharge rates increased from about 225 gpm to 1,800 gpm ([2]). In practice, factors such as depth, pump characteristics, layout, etc., affect the question of well size more than capacity, and generally speaking, it costs less money to put down and equip two medium-sized wells (8-16 inches) than one large well (24-36 inches and larger); and the combined yield of the smaller wells is almost always greater than the yield of the larger well.

Jacob ([1]) did not mention partial penetration, but it is obvious that the efficiency of a well depends on the degree it penetrates an aquifer. The partial penetration increases the drawdown in a well because some of the water that enters the well must percolate upward from the materials beneath the well or downward from the materials above the screen or the perforations. Water percolating vertically to a well moves a greater distance than if it had percolated horizontally and across planes of greater resistance (i.e. horizontal permeability is greater than vertical permeability), and thus more drawdown occurs than would occur had the well completely penetrated the aquifer. For partially penetrating wells, Rorabaugh ([2]) states that step-drawdown test results are satisfactory for constant-time problems, and can be used for variable-time problems under certain conditions. Walton ([6]) shows that the efficiency of a well varies directly with the amount of penetration and that the efficiency of a well is least affected by partial penetration of aquifers having a high transmissibility.

6.2. *Well Development Factor*

The well-loss constant, C, is empirically derived and depends on the effective open area of the well screen or perforated casing as indicated above, and on the condition of the well screen or face and the effectiveness of development of the well. By "development" is meant the removal of the silt and fine sand around a well screen so as to produce a natural filter of coarser and more uniform sand or gravel which in turn provides the greatest amount of open space for water to flow through. The larger these openings, the less the velocity; the less velocity, the less friction; the less friction, the greater the efficiency. The effectiveness of development generally can be appraised from the results of a step-drawdown test.

Table I lists numerical values for the well-loss constant, C, for the 31 production wells, and these range from about 10^{-3} to 10^{-6} (ft/gpm^2; converted to Jacob's notation sec^2/ft^5 by multiplying by the conversion factor 2×10^5). Shown also is the pumping rate for which the well was designed, and the corresponding efficiency (BQ/s_w) at this required rate. These values are plotted on Figures 5-8 as points on the curves and show at a glance the relationship to the 50 per cent efficiency line — that point at which the total drawdown in the well is divided evenly between formation loss and well loss.

As stated previously, the production wells in Iran were subjected to lengthy development work which continued until sand-free water was produced. It is thus assumed that the wells, as designed, were properly developed. Unfortunately, the advantages of this development may in part be offset by poor design or improper choice of screen. Wells with "low" efficiency may fall in this category, and for the purpose of this paper, wells with a pumping efficiency less than 50 per cent are thus categorized.

There is a tendency to ascribe the low efficiency of certain wells to various selective "causes" such as inadequate screen length, too-small screen openings, partial penetration and insufficient development. Realizing that the step-drawdown tests are an after-the-fact procedure and that each results in data that are applicable only to the particular well, it appears that to be prudent it is best to avoid suggesting specific reasons to explain a given performance.

The last column of Table 1 gives the ratio of well loss to formation loss times 100; the resulting figure for each supply well suggesting an empirical grading of the effectiveness of well development. Thus, values of C/B x 100 less than 0.1 suggest "very effective" development; values of 0.1-0.5 indicate "effective" development; values 0.5-1.0 indicate "fairly effective" development; and values greater than 1.0 indicate "poorly effective" development. (Or perhaps the effectiveness of development should be graded "excellent", "good", "fair", and "poor".) The fact that certain wells of "low" efficiency as shown in Figures 5-8 have development values in the "effective" range 0.1-0.5 presents no ambiguity if it is remembered that other "causes" are contributing to the drawdown in the wells.

Walton ([6]) suggests that the degree of well deterioration subsequent to use be appraised using values of the well-loss constant, C, as criteria. Thus, the value of C of a properly developed stable well is generally less than 5×10^{-6} ft/gpm^2; (or 1 sec^2/ft^5); values of C between 1 and 10 sec^2/ft^5 indicate mild deterioration as screen slots become clogged after heavy pumping; and when C is greater than 10 sec^2/ft^5 clogging is severe. Consequently, if step-tests are run on production wells after a lapse of time, it can be determined whether or not the well has deteriorated and if rehabilitation is required.

7. Summary of findings

The performance of a well may be evaluated through graphical analysis of the approximate equation $s_w = BQ + CQ^2$ where s_w is the total drawdown in the well, BQ represents the drawdown due to formation loss, and CQ^2 represents the drawdown due to well loss. The efficiency of the well may then be defined as BQ/s_w.

The efficiency of a well is governed largely by the magnitude of well loss and thus falls off rapidly as discharge is increased. The efficiency of a well in an aquifer having a high transmissibility is affected by well loss to a greater degree than the efficiency of a well in an aquifer having a low transmissibility, and it is least affected by partial penetration of aquifers having a large transmissibility.

In a given location, a new well may be brought to optimum efficiency by screening as much of the thickness of the aquifer as is practical with as large an area of openings as is consistent with the natural gradation of the formation. This slot area may be increased by using a longer screen, a greater diameter screen, or a screen having a greater per cent of open area with respect to the total surface area of the screen cylinder. Maximum efficiency can then be obtained only if development work is sufficient to remove fine materials from the well face and adjacent formation to produce a stabilized, graded, natural gravel pack which results in a zone of greater permeability about the well. This situation not only reduces the entrance velocity of water but also prevents the migration of fine materials into the well and precludes sand pumping.

The effectiveness of well development may be appraised by comparing the well loss to the formation loss. The degree of deterioration of an old well may be appraised by comparison of well-loss values, or preferably, by comparing steptest results obtained when the well was first completed and then after the well has been producing for a period of time.

Through the accumulation and presentation of empirical data, the stepdrawdown test procedure should aid greatly in understanding the performance characteristics of wells — due consideration being given to the transient behavior of the aquifer. An appreciation of the factors affecting well efficiency can thus lead to savings in construction costs and operating costs.

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TABLE I

TABLE 1

Well data and step drawdown test data

Well Number	Effective open area			Development Work	Pumping test results				Well efficiency		
	Screen (S) or Perforated Casing (P)				$s_w = BQ + CQ^2$		Transmissibility	Permeability	Required Rate	Pumping efficiency	Development Factor
	Length (Ft)	Slot Size or No of Perforations	Area (Sq. In)	(Hours)	B	C	(GPD/Ft)	(GPD/Ft^2)	(GPM)	(BQ/s_w)	$C/B \times 100$
K - 10	17	(P) 68	204	0	0.0544	0.000026	34,000	400	—	—	—
S8-19	15	(P) 60	180	0	0.060	0.0022	80,000	500	—	—	—
S8-19	15	(P) 128	308	0	0.073	0.000062	80,000	500	—	—	—
1-18	100	(P) 228	428	0	0.0122	0.00048	75,000	450	—	—	—
2-3	15	(P) 60	180	0	0.023	0.00018	92,000	575	—	—	—
8-17	15	(P) 60	180	0	0.0125	0.00015	78,000	490	—	—	—
Ham-7	30	(S) No. 80	5,220	78	0.093	0.000055	8,100	510	600	74%	0.059
Sar-2	30	(S) Nos. 60,40	4,700	81	0.248	0.00016	7,400	85	200	89	.065
Sar-3	30	(S) Nos. 60,40	4,700	99	0.380	0.00057	9,400	85	150	82	.15
Kha-4	30	(S) No. 60	5,100	80	0.300	0.00045	14,800	500	350	66	.15
Ham-4	40	(S) Nos. 80, 20, 10	4,670	84	0.130	0.00022	5,600	430	550	53	.17
Mar-2	10	(S) No. 60	1,700	28	0.137	0.00077	45,000	700	150	54	.56
Ham-6	30	(S) No. 80	5,220	70	0.420	0.0027	3,000	130	170	48	.64
Nag-7	10	(S) No. 60	1,700	86	0.185	0.00167	5,800	390	55	67	.90
26-15	90	(P) 190	570	0	0.044	0.000092	67,000	1,500	—	—	—

TABLE 1 (continued)

Well data and step drawdown test data

Well Number	Effective open area: Screen (S) or Perforated Casing (P)			Development Work	Pumping test results: $s_w = BQ + CQ^2$		Pumping test results		Well efficiency		
	Length (Ft)	Slit Size or No of Perforations	Area (Sq. In)	(Hours)	B	C	Transmissability (GPD/Ft)	Permeability (GPD/Ft^2)	Required Rate (GPM)	Pumping efficiency (BQ/s_w)	Development Factor $C/B \times 100$
26-15	95	(P) 286	666	0	0.037	0.000012	67,000	1,500	—	—	—
Aja-2	25	(S) No. 80	3,480	89	0.038	0.000030	32,000	840	400	76	.079
31-53	117	(S) No. 35	14,040	24	0.0134	0.000012	108,000	900	—	—	—
Che-1	10	(S) No. 40	1,300	128	0.0205	0.000019	64,000	1,400	300	78	.093
Kus-2	20	(S) No. 60	3,400	52	0.126	0.000140	81,000	950	300	75	.11
Aja-1	20	(S) No. 80	3,480	72	0.018	0.000034	37,000	920	400	57	.19
Ush-5	20	(S) No. 60	3,480	93	0.093	0.000270	204,000	1,400	400	46	.29
Ker-2	50	(S) Nos. 80,60,10	7,260	224	0.100	0.00056	90,000	1,400	240	43	.56
33-56	124	(S) No. 20	9,548	24	0.009	0.00018	155,000	1,700	—	—	—
Nag-8	15	(S) Nos. 60, 40	2,350	134	0.015	0.00090	18,000	1,200	125	12	6.0
14-27	30	(P) 60	180	0	0.00086	0.000012	530,000	2,400	—	—	—
Kus-1	20	(S) No. 80	3,480	73	0.0415	0.000018	142,000	2,400	500	82%	0.43
Teh-1	20	(S) No. 60	3,400	72	0.0052	0.0000062	610,000	4,500	150	85	.12
Aha-2	15	(S) No. 40	1,950	103	0.125	0.00016	88,000	3,000	250	76	.13
Teh-2	20	(S) No.80	3,480	72	0.0056	0.0000080	660,000	4,500	150	82	.14

TABLE 1 (continued)

Teh-2	20	(S) No.80	3,480	72	0.0036	0.0000080	660,000	4,500	150	[illegible]	[illegible]

TABLE 1 (continued)

Well data and step drawdown test data

Well Number	Effective open area Screen (S) or Perforated Casing (P)			Development Work	Pumping test results						
	Length (Ft)	Slot Size or No of Perforations	Area (Sq. In)	(Hours)	$s_w = BQ + CQ^2$ B	C	Transmissability (GPD/Ft)	Permeability (GPD/Ft^2)	Required Rate (GPM)	Pumping efficiency (BQ/s_w)	Development Factor $C/B \times 100$
Mar-1	10	(S) No. 100	1,800	101	0.071	0.00018	97,000	2,500	180	69	.25
Ghu-3	15	(S) No. 80	2,610	111	0.013	0.000034	130,000	8,700	300	56	.26
Ker-4	30	(S) Nos. 60, 40	4,700	79	0.230	0.0019	95,000	2,000	100	53	.83
Ghu-4	15	(S) Nos. 80, 30	1,970	109	0.0075	0.000065	165,000	4,700	300	28	.86
Kha-3	10	(S) No. 60	1,700	136	0.045	0.00038	63,000	6,300	350	25	.84
Dez-4	20	(S) No. 100	3,600	50	0.0095	0.00009	480,000	3,200	300	26	.95
Nau-2	20	(S) No. 60	3,400	119	0.080	0.00074	163,000	5,400	100	52	.93
Nau-3	20	(S) Nos. 80, 60	3,440	96	0.012	0.00012	145,000	6,600	100	50	1.00
Dez-3	15	(S) No. 100	2,700	100	0.010	0.00017	440,000	3,000	300	16	1.7
Nag-9	10	(S) No. 80	1,740	76	0.010	0.00295	22,000	2,200	95	3	29.5
65-50	55	(P) 220	660	0	0.0115	0.000041	480,000	13,700	—	—	—
55-51	45	(P) 540	1,620	0	0.00082	0.000004	3,000,000	66,700	—	—	—
31-30	67	(P) 134	402	0	0.00115	0.0000085	1,850,000	53,000	—	—	—
24-33	20	(P) 130	390	0	0.00022	0.0000032	2,900,000	64,500	—	—	—
55-52	45	(S) No. 60	7,640	36	0.00122	0.00000017	3,000,000	66,700	—	—	—
Mia-3	20	(S) Nos. 60, 40	3,000	56	0.0074	0.0000042	440,000	20,000	500	78	.057
Mia-4	20	(S) Nos. 80, 40	3,040	101	0.0110	0.000010	660,000	30,000	500	70	.091